13th European Drag Reduction Meeting, Aussois, France, 1-4 June 2004

OPTIMAL ROTARY CONTROL OF THE CYLINDER WAKE USING POD REDUCED ORDER MODEL

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Abstract

This communication investigates the optimal control approach for the active control and drag optimization of incompressible viscous flow past cylinders. The control function is the time harmonic angular velocity of the rotating cylinder. The wake flow is solved in the laminar regime ($Re = 200$) with a finite element method. Due to the CPU and memory costs related to the optimal control theory, a *Proper Orthogonal Decomposition* (POD) Reduced Order Model (ROM) is used as the state equation. Since the POD basis represents only velocities, we minimize a drag-related cost function characteristic of the wake unsteadiness. The optimization problem is solved using Lagrange multipliers to enforce the constraints. 25% of relative drag reduction is found when the Navier-Stokes equations are controlled using the optimal control function determined with the POD ROM. A cost reduction factor of respectively one hundred and six hundred is obtained for respectively the CPU time and the memory.

1 Optimal control based on POD ROM

Different experimental or numerical approaches have been successfully employed for the control of a wake flow but recently optimal control theory attracted increased attention. However the numerical costs (CPU and memory) associated with the adjoint equation-based methods used to solve these optimization problems are so important that the three-dimensional Navier-Stokes equations are rarely studied. For cutting down the numerical costs different approaches are possible. One promising approach is to first use reduced order models to approximate the fluid flow and then to optimize exactly the reduced models. In this study, a reduced order model based on Proper Orthogonal Decomposition (POD) is developed. The velocity expansion on the POD basis functions $\{\boldsymbol{\phi}^{(k)}\}_{k=1}^{N_{POD}}$ writes

$$\boldsymbol{u}(\boldsymbol{x},t) = \boldsymbol{u}_m(\boldsymbol{x}) + \gamma(t)\,\boldsymbol{u}_c(\boldsymbol{x}) + \sum_{k=1}^{N_{POD}} a^{(k)}(t)\boldsymbol{\phi}^{(k)}(\boldsymbol{x}) \tag{1}$$

where $\boldsymbol{u}_m(\boldsymbol{x})$ is the mean velocity field obtained as an ensemble average of the flow realizations, γ is the unsteady tangential velocity of the cylinder and $\boldsymbol{u}_c(\boldsymbol{x})$ is an arbitrary control function satisfying homogeneous boundary conditions. A convenient way to generate it is to take the solution for the steady cylinder rotation with $\gamma = 1$. Inserting the expansion (1) into the Galerkin projection of the Navier-Stokes equations onto the POD functions, the reduced order control model (POD ROM) is obtained:

$$\frac{d\,a^{(i)}(t)}{d\,t} = \mathcal{A}_i + \sum_{j=1}^{N_{gal}} \mathcal{B}_{ij}\,a^{(j)}(t) + \sum_{j=1}^{N_{gal}}\sum_{k=1}^{N_{gal}} \mathcal{C}_{ijk}\,a^{(j)}(t)a^{(k)}(t) + \mathcal{D}_i\,\frac{d\,\gamma}{d\,t} + \left(\mathcal{E}_i + \sum_{j=1}^{N_{gal}} \mathcal{F}_{ij}\,a^{(j)}(t)\right)\gamma + \mathcal{G}_i\gamma^2 \tag{2}$$

The rotation rate $\gamma(t)$ is then determined using an optimal control approach. Our first objective is to minimize the flow drag. However, since only the flow velocities are directly represented by the POD basis functions, we need to introduce a drag-related cost function. A natural control aim is the reduction of the wake unsteadiness i.e. the energy of the wake. Mathematically, this goal is expressed as the following functional

$$\mathcal{J}(\boldsymbol{a},\gamma(t)) = \int_0^T J(\boldsymbol{a},\gamma(t))\,dt = \frac{\alpha}{2}\int_0^T \sum_{i=1}^{N_{gal}} \left(a^{(i)}(t)\right)^2\,dt + \frac{\beta}{2}\int_0^T \gamma(t)^2\,dt. \tag{3}$$

The optimality system composed by the state equation (2) and the equations (4) and (5) representing respectively the adjoint equation and the optimality condition is determined using Lagrange multiplier methods:

$$\frac{d\,\xi^{(i)}(t)}{dt} = -\alpha a^{(i)}(t) - \sum_{j=1}^{N_{gal}} \left(\mathcal{B}_{ji} + \gamma(t)\,\mathcal{F}_{ji} + \sum_{k=1}^{N_{gal}} \left(\mathcal{C}_{jik} + \mathcal{C}_{jki}\right) a^{(k)}(t) \right) \xi^{(j)}(t), \quad \xi^{(i)}(T) = 0, \tag{4}$$

$$\delta\gamma(t) = -\sum_{i=1}^{N_{gal}} \mathcal{D}_i \frac{d\xi^{(i)}}{dt} + \beta\gamma + \sum_{i=1}^{N_{gal}} \left(\mathcal{E}_i + \sum_{j=1}^{N_{gal}} \mathcal{F}_{ij} a^{(j)} + 2\mathcal{G}_i \gamma(t) \right) \xi^{(i)}. \tag{5}$$

In this study, the POD low order model (2) is not reset during the optimization process. Clearly, it corresponds to the assumption that one must be able to predict the system behaviour during the period T of the optimization, hence the importance of developing accurate low order models. Hence, we derived generalized POD basis functions by imposing slowly varying amplitude and frequency sinusoid as rotation rate for the cylinder. This temporal excitation γ_e is mathematically represented by the function:

$$\gamma_e(t) = A_1 \sin(2\pi S_{t1}\, t) \times \sin(2\pi S_{t2}\, t - A_2 \sin(2\pi S_{t3}\, t))$$

where $A_1 = 4$, $A_2 = 18$, $S_{t1} = 1/120$, $S_{t2} = 1/3$ and $S_{t3} = 1/60$.

The optimality system (2), (4) and (5) is then iteratively solved, from an initial guess γ_e with a conjugate gradient method. After convergence corresponding to 40% of reduction for the wake unsteadiness, the optimal control γ_{opt} (see figure (1)) is obtained. This optimal control law can be approximated by an harmonic function $\gamma_{opt}(t) = A\sin(2\pi S_t t)$ with $A = 2.2$ and $S_t = 0.53$.

By definition of the optimization problem, the control function γ_{opt} is optimal for the POD ROM. However there is no mathematical proof of optimality with respect to the Navier-Stokes model and the initial objective of this study is the optimal reduction of drag. Therefore it is necessary to solve the Navier-Stokes equations with a rotary control defined by $\gamma(t) = \gamma_{opt}(t)$ to determine the effect of this control function on the drag coefficient.

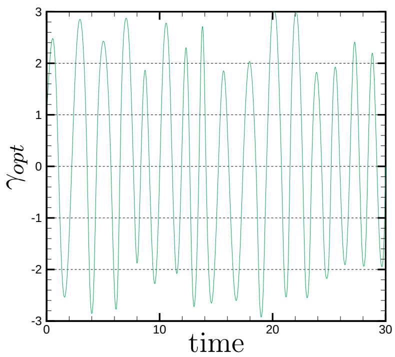

Figure 1: Time history of the optimized control function γ_{opt}.

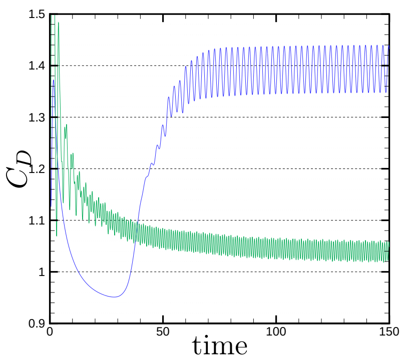

Figure 2: Comparison of the time evolution of drag of the uncontrolled (blue) and controlled (green) flow.

When the optimal control law $\gamma_{opt}(t)$ is applied to the cylinder, the mean drag coefficient is reduced from a value approximatively equal to 1.4 to a value equal to 1.04 (see figure (2)).

2 Conclusions

When the cylinder wake is controlled using the function γ_{opt}, an important relative drag reduction is obtained (more than 25%). Comparing to studies where the two dimensional Navier-Stokes equations are used to solve the optimal control problem, slightly lower value of drag reduction is found (25% compared to 30%). The main advantage of our approach is that the numerical costs (CPU and memory) are negligible (of the order of 1%).