

# Control of the cylinder wake in the laminar regime by Trust-Region methods and POD Reduced Order Models.

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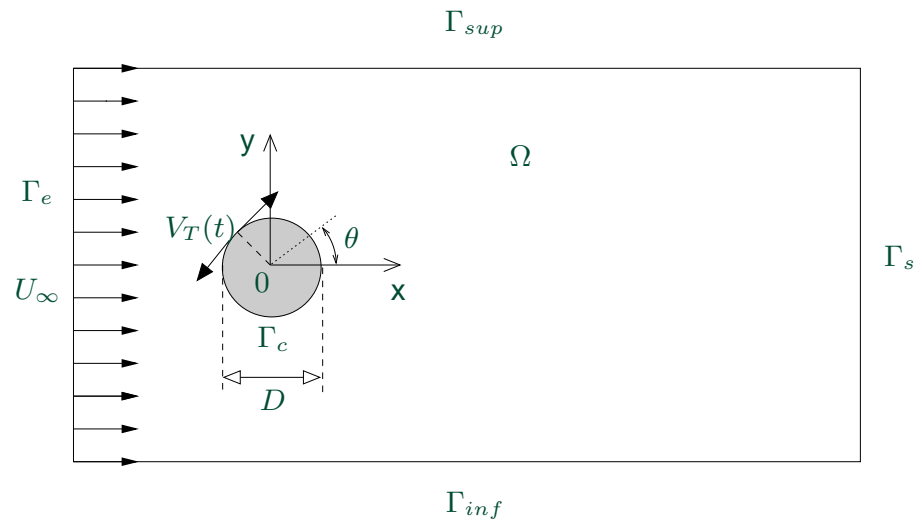
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# Introduction Configuration and numerical method

- Two dimensional flow around a circular cylinder at  $Re = 200$
- Viscous, incompressible and Newtonian fluid
- Cylinder oscillation with a tangential sinusoidal velocity  $\gamma(t)$

$$\gamma(t) = \frac{V_T}{U_\infty} = A \sin(2\pi St_f t)$$

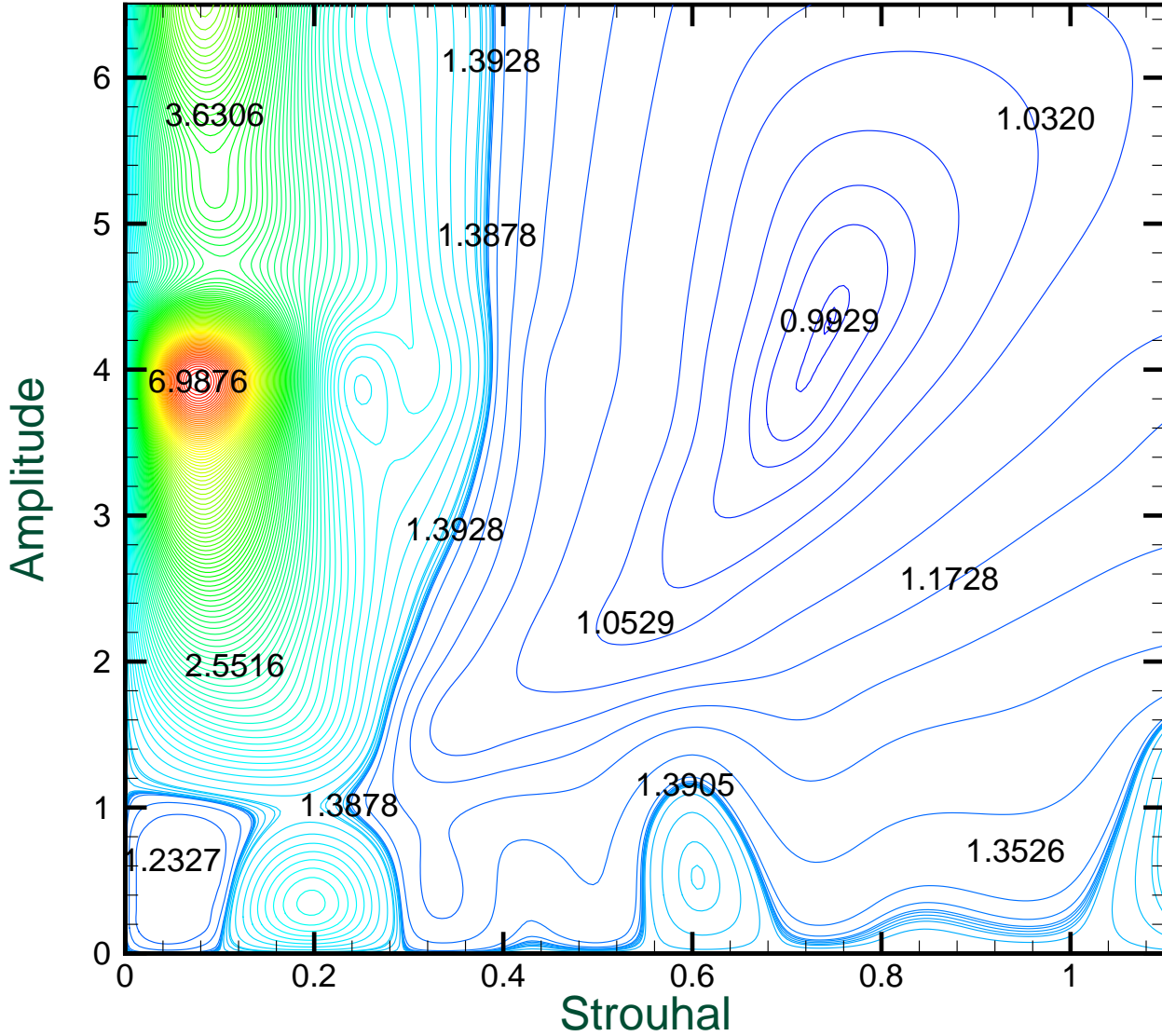


**Find the control parameters  $c = (A, St_f)^T$  such that the mean drag coefficient is minimized**

$$\langle C_D \rangle_T = \frac{1}{T} \int_0^T \int_0^{2\pi} 2 p n_x R d\theta dt - \frac{1}{T} \int_0^T \int_0^{2\pi} \frac{2}{Re} \left( \frac{\partial u}{\partial x} n_x + \frac{\partial u}{\partial y} n_y \right) R d\theta dt,$$



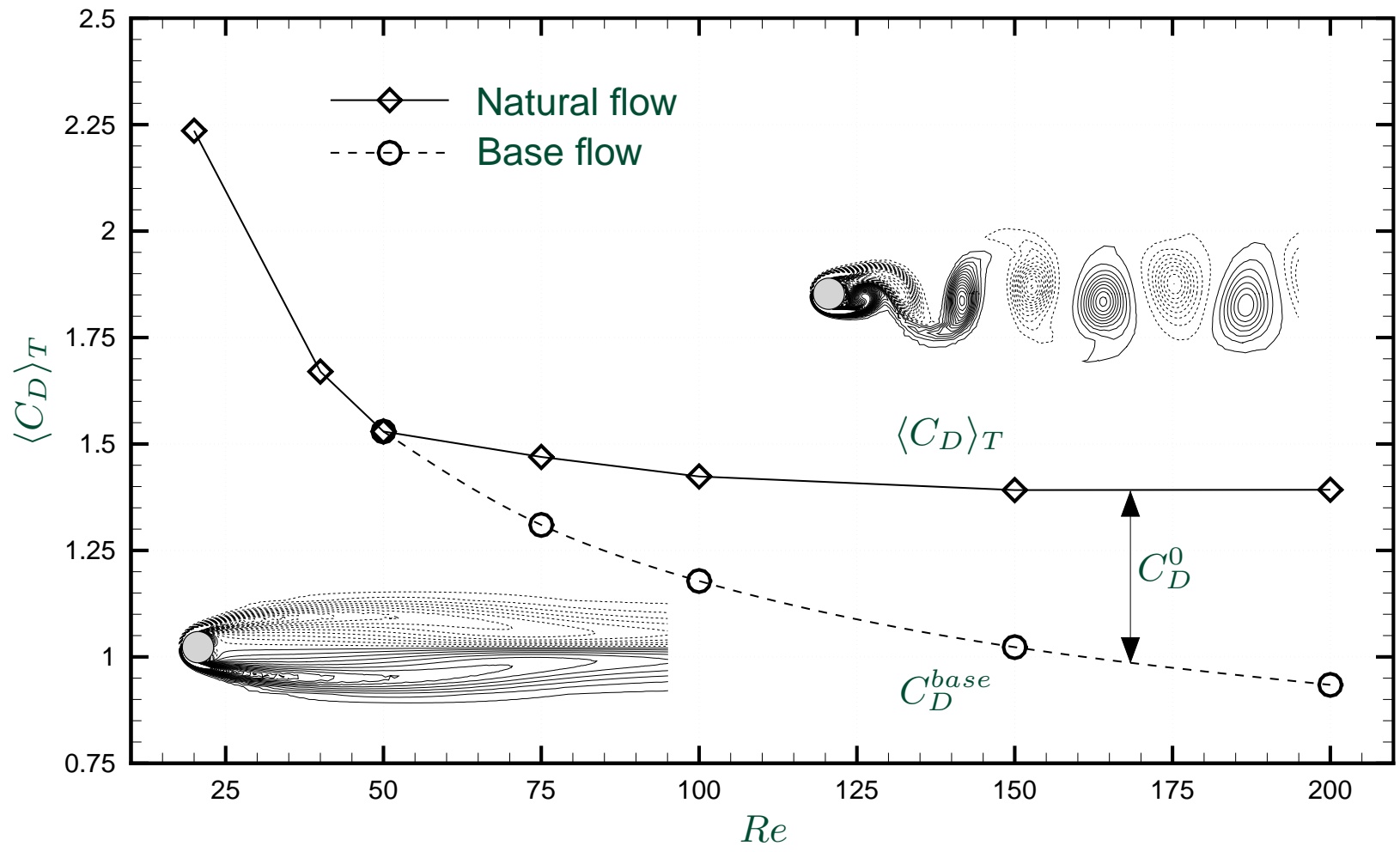
# Introduction *Parametric study*



Variation of the mean drag coefficient with  $A$  and  $St_f$ .  
Numerical minimum  $(A_{min}, St_{f_{min}}) = (4.3, 0.74)$ .



# Introduction *Mean drag coefficient & steady unstable base flow*

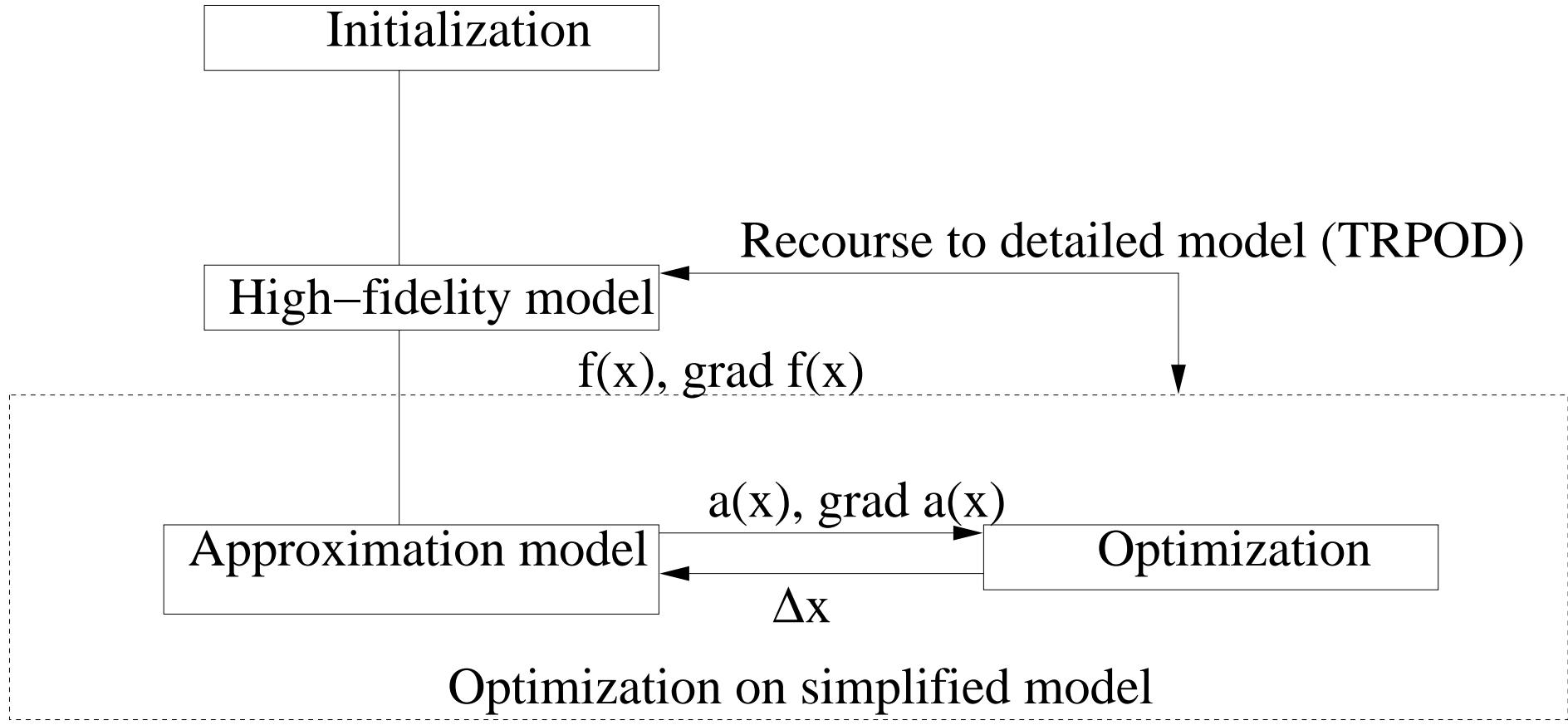


**Fig. :** Variation with the Reynolds number of the mean drag coefficient. Contributions and corresponding flow patterns of the base flow and unsteady flow.

Protas, B. et Wesfreid, J.E. (2002) : Drag force in the open-loop control of the cylinder wake in the laminar regime. *Phys. Fluids*, **14**(2), pp. 810-826.



# Reduced Order Model (ROM) and optimization problems



# Reduced Order Model (ROM) Proper Orthogonal Decomposition (POD)

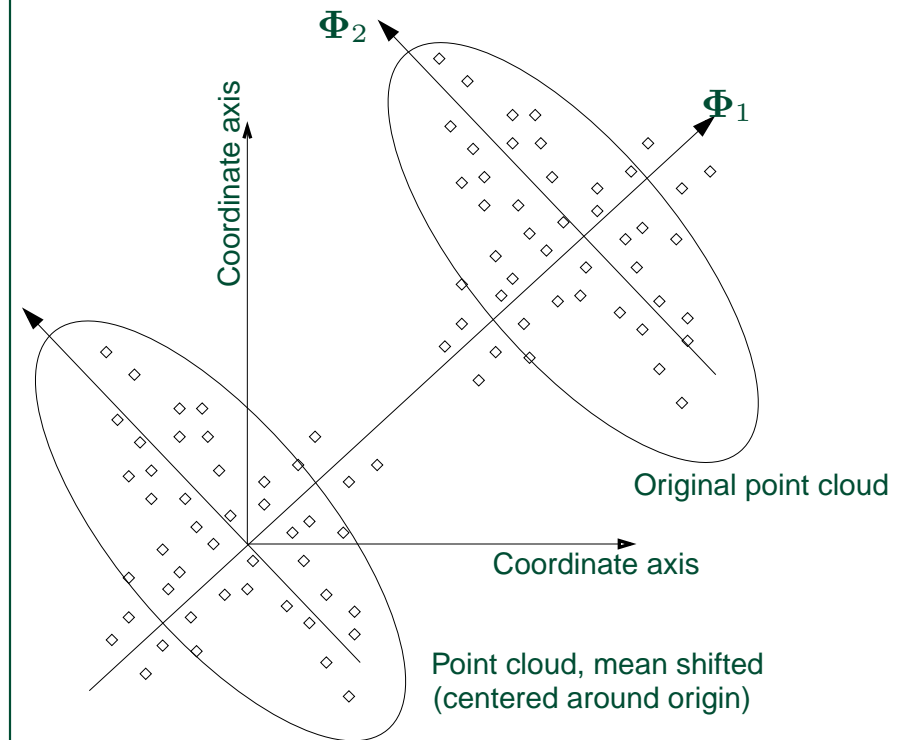
- ▶ Introduced in turbulence by Lumley (1967).
- ▶ Method of information compression
- ▶ Look for a realization  $\Phi(\mathbf{X})$  which is closer, in an average sense, to realizations  $\mathbf{u}(\mathbf{X})$ . ( $\mathbf{X} = (\mathbf{x}, t) \in \mathcal{D} = \Omega \times \mathbb{R}^+$ )
- ▶  $\Phi(\mathbf{X})$  solution of the problem :

$$\max_{\Phi} \langle |(\mathbf{u}, \Phi)|^2 \rangle \quad \text{s.t.} \quad \|\Phi\|^2 = 1.$$

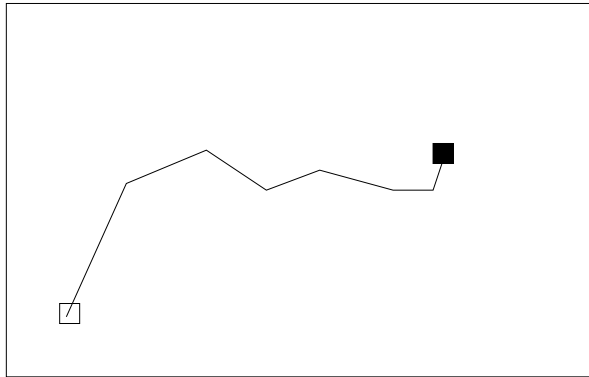
- ▶ Snapshots method, Sirovich (1987) :

$$\int_T C(t, t') a^{(n)}(t') dt' = \lambda^{(n)} a^{(n)}(t).$$

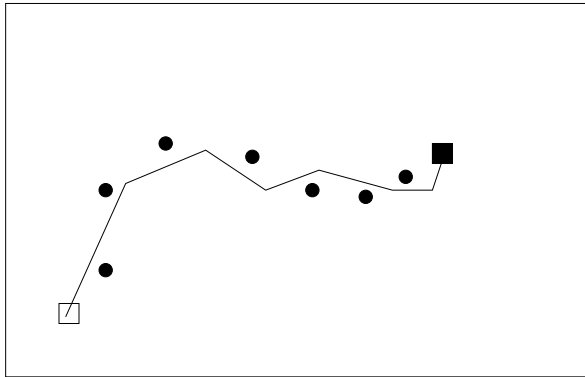
- ▶ Optimal convergence in  $L^2$  norm (energy) of  $\Phi(\mathbf{X})$   
 $\Rightarrow$  Dynamical order reduction is possible.



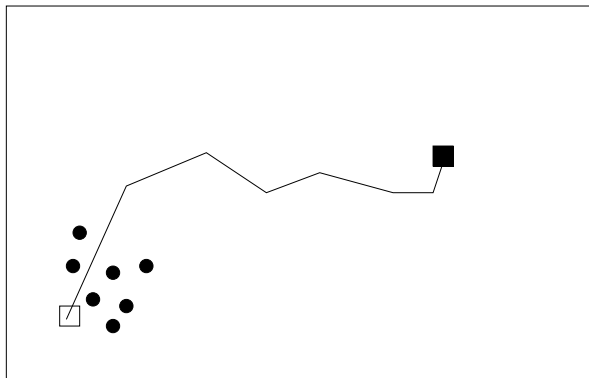
# ROM Parameter sampling in an optimization setting



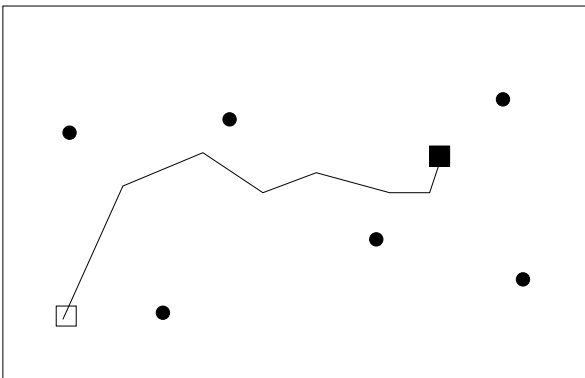
General configuration.



Ideal sampling.



Unsuitable sampling.



Unsuitable sampling.

Discussion of parameter sampling in an optimization setting (from Gunzburger, 2004).

— path to optimizer using full system, □ initial values, ■ optimal values, and ● parameter values used for snapshot generation.



# ROM A simple configuration, a rich dynamical behavior

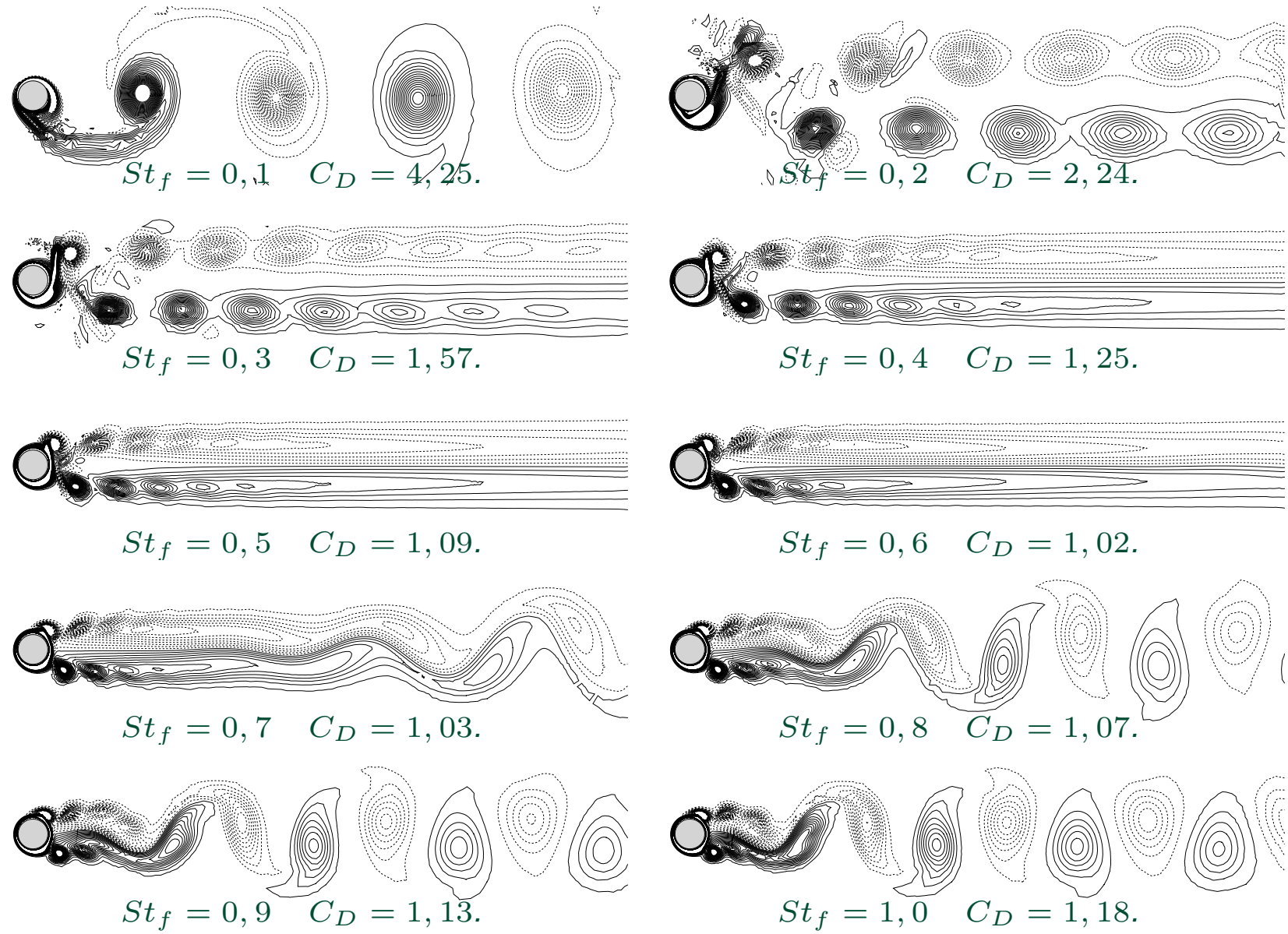


Fig. : Iso-values of the vorticity fields  $\omega_z$  for  $A = 3$





- Necessity for a given reference flow to introduce new modes : either new operating conditions or shift-modes

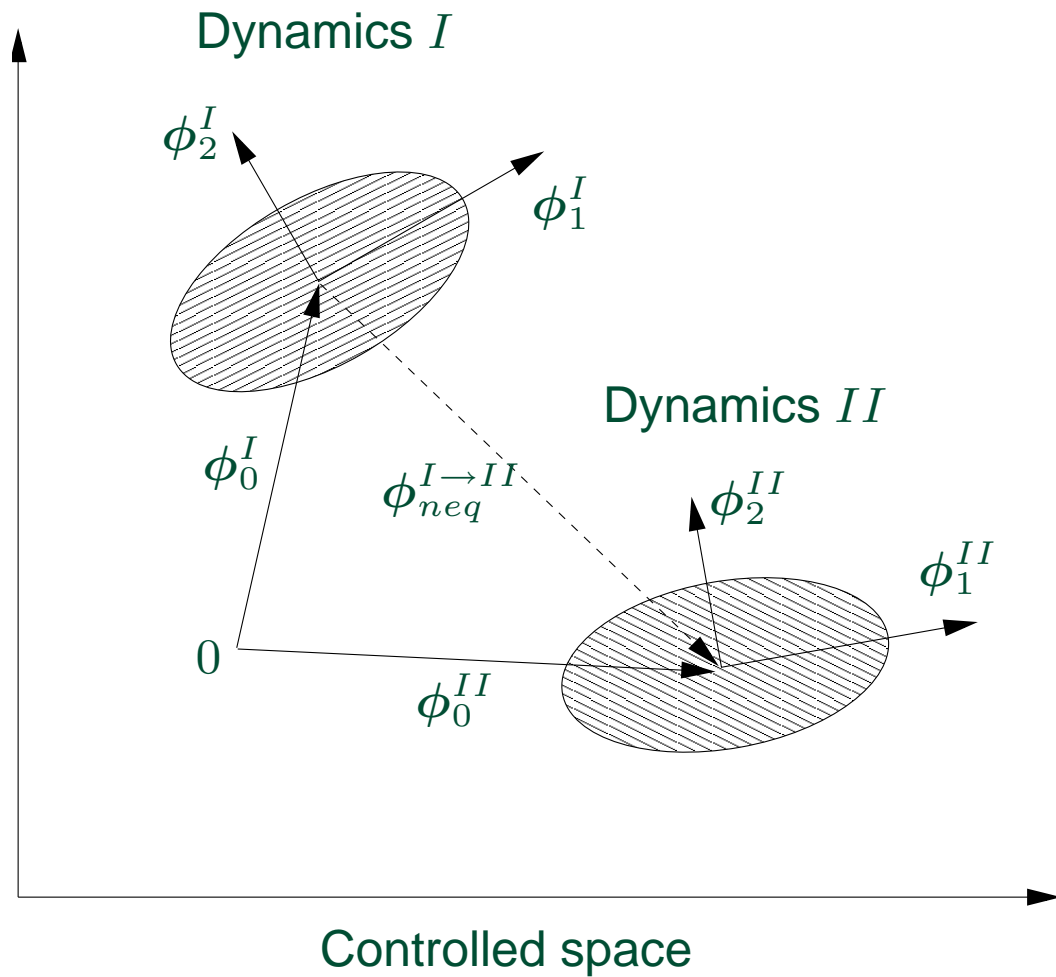


Fig. : Schematic representation of a dynamical transition with a non-equilibrium mode



# ROM A robust POD surrogate for the drag coefficient

► POD approximations consistent with our approach :

$$\mathbf{U}(\mathbf{x}, t) = (u, v, p)^T = \underbrace{\sum_{i=0}^N a_i(t) \phi_i(\mathbf{x})}_{\text{Galerkin modes}} + \underbrace{\sum_{i=N+1}^{N+M} a_i(t) \phi_i(\mathbf{x})}_{\text{non-equilibrium modes}} + \underbrace{\gamma(\mathbf{c}, t) \mathbf{U}_c(\mathbf{x})}_{\text{control function}}$$

Physical aspects	Modes	Dynamical aspects
<b>actuation mode</b>	$U_c$	predetermined dynamics
<b>mean flow mode</b>	$U_m, i = 0$	$a_0 = 1$
<b>Galerkin modes</b> Dynamics of the reference flow $I$	$i = 1$	<b>POD ROM</b> Temporal dynamics of the modes (eventually, the mode $i = 0$ is solved then $a_0 \equiv a_0(t)$ )
	$i = 2$	
	$\dots$	
	$i = N$	
<b>non-equilibrium modes</b> Inclusion of new operating conditions $II, III, IV, \dots$	$i = N + 1$	
	$\dots$	
	$i = N + M$	



- Galerkin projection of *NSE* onto the POD basis :

$$\left( \phi_i, \frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot (\mathbf{u} \otimes \mathbf{u}) \right) = \left( \phi_i, -\nabla p + \frac{1}{Re} \Delta \mathbf{u} \right).$$

- Reduced order dynamical system where only  $(N + M + 1)$  ( $\ll N_{POD}$ ) modes are retained (state equations) :

$$\begin{aligned} \frac{d a_i(t)}{d t} &= \sum_{j=0}^{N+M} \mathcal{B}_{ij} a_j(t) + \sum_{j=0}^{N+M} \sum_{k=0}^{N+M} \mathcal{C}_{ijk} a_j(t) a_k(t) \\ &\quad + \mathcal{D}_i \frac{d \gamma}{d t} + \left( \mathcal{E}_i + \sum_{j=0}^{N+M} \mathcal{F}_{ij} a_j(t) \right) \gamma(\mathbf{c}, t) + \mathcal{G}_i \gamma^2(\mathbf{c}, t), \\ a_i(0) &= (\mathbf{U}(\mathbf{x}, 0), \phi_i(\mathbf{x})). \end{aligned}$$

$\mathcal{B}_{ij}$ ,  $\mathcal{C}_{ijk}$ ,  $\mathcal{D}_i$ ,  $\mathcal{E}_i$ ,  $\mathcal{F}_{ij}$  and  $\mathcal{G}_i$  depend on  $\phi_i$ ,  $U_c$  and  $Re$ .



# Surrogate drag function and model objective function *Generalities*

- Drag operator :

$$\mathcal{C}_D : \mathbb{R}^3 \rightarrow \mathbb{R}$$
$$\mathbf{u} \mapsto 2 \int_0^{2\pi} \left( u_3 n_x - \frac{1}{Re} \frac{\partial u_1}{\partial x} n_x - \frac{1}{Re} \frac{\partial u_1}{\partial y} n_y \right) R d\theta, \quad (1)$$

- Surrogate drag function :

$$\widetilde{C}_D(t) = \underbrace{a_0(t)N_0 + \sum_{i=N+1}^{N+M} a_i(t)N_i}_{\text{evolution of the mean drag}} + \underbrace{\sum_{i=1}^N a_i(t)N_i}_{\text{fluctuations } C'_D(t)} \quad \text{with } N_i = \mathcal{C}_D(\phi_i).$$

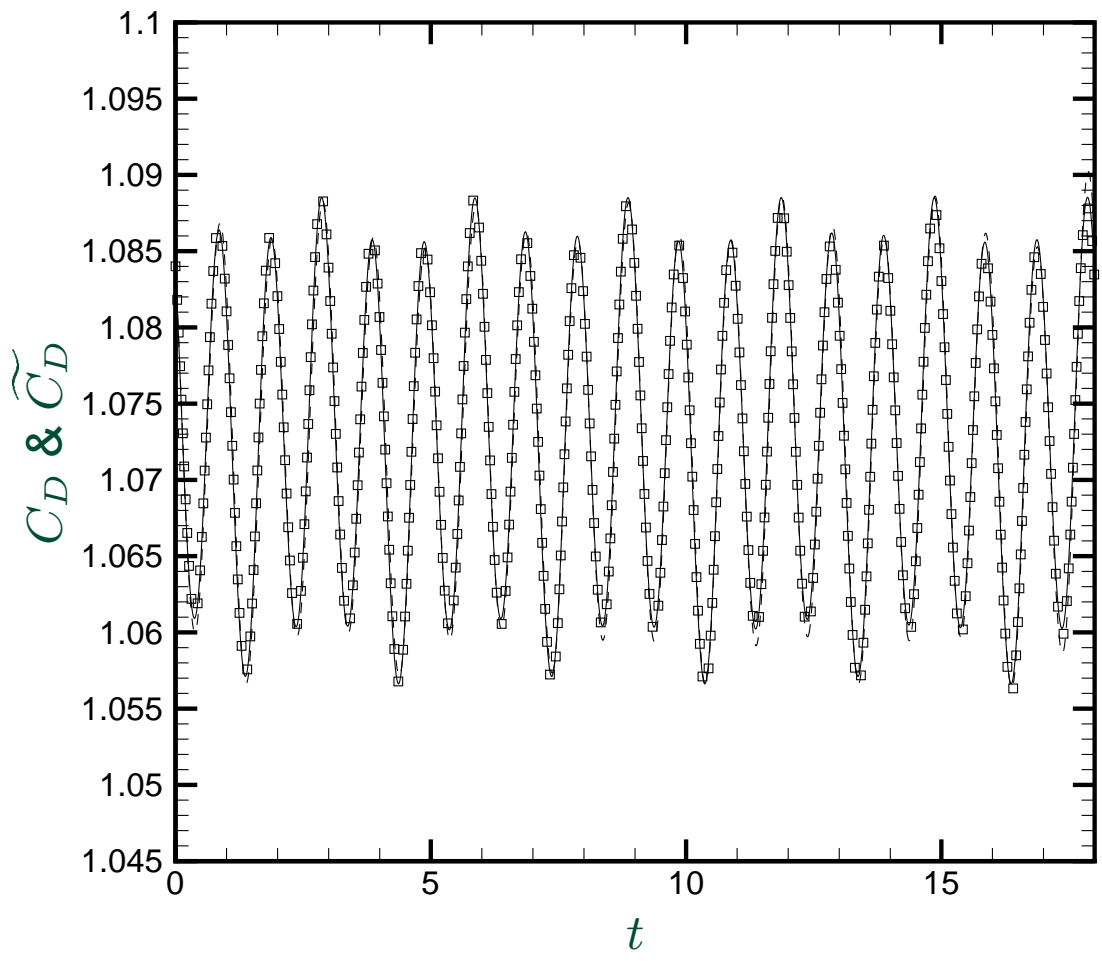
- Model objective function :

$$m = \langle \widetilde{C}_D(t) \rangle_T = \frac{1}{T} \int_0^T \left( a_0(t)N_0 + \sum_{i=N+1}^{N+M} a_i(t)N_i \right) dt.$$

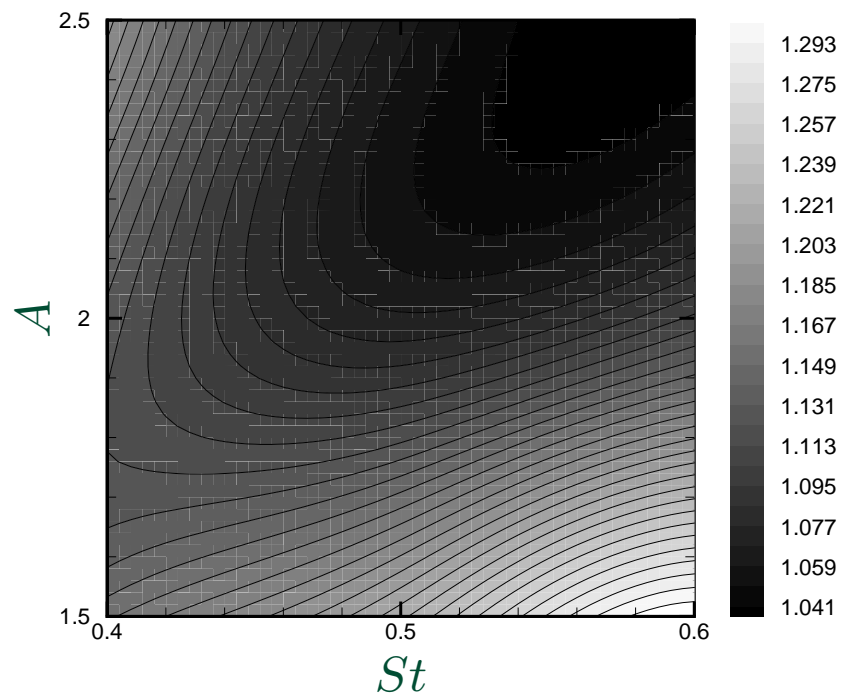


# Surrogate drag function *Test case $A = 2$ and $St = 0.5$*

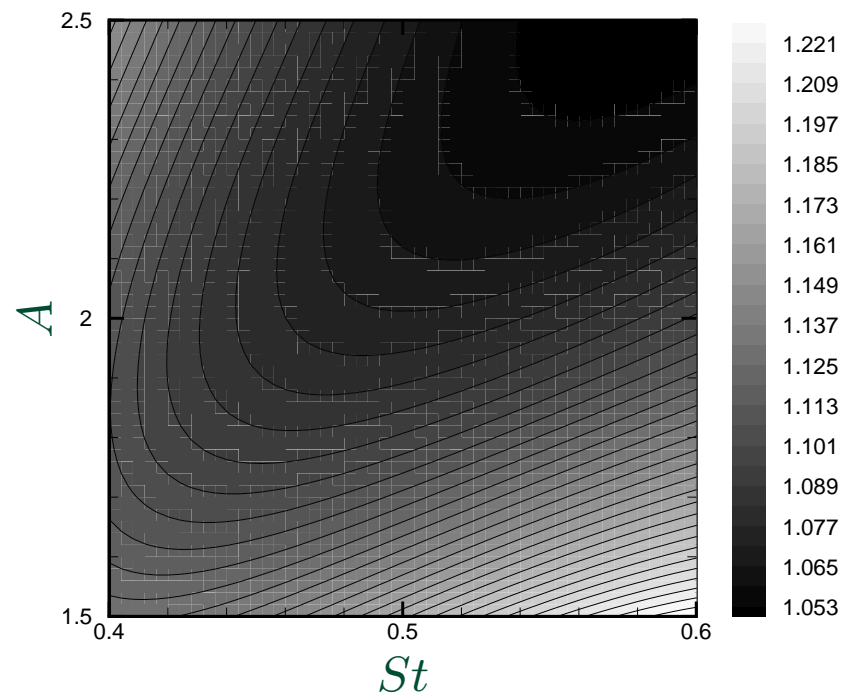
- Comparison of real drag coefficient  $C_D$  (symbols) and model function  $\tilde{C}_D$  (lines) at the design parameters.



# Robustness of the model objective function *Test case $A = 2$ and $St = 0.5$*



(a) Real objective function  $\mathcal{J}$



(b) Model objective function  $m$

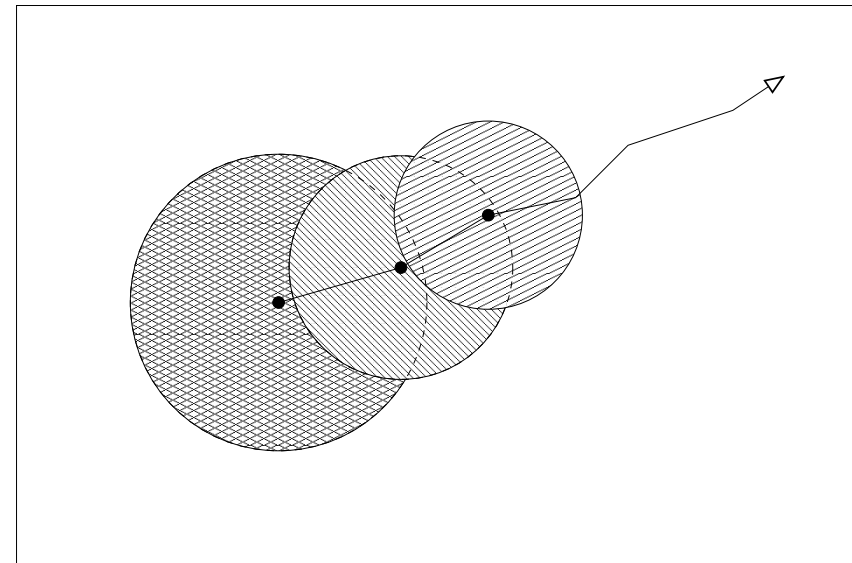
Fig. : Comparison of the real and the model objective functions associated to the mean drag coefficient.



Range of validity of the POD ROM restricted to a vicinity of the design parameters

Objective : Use ROMs to solve large-scale optimization problems with assurance of :

1. Automatic restriction of the range of validity
2. Global convergence



Solution

- Embed the POD technique into the concept of trust-region methods :  
**Trust-Region Proper Orthogonal Decomposition (Fahl, 2000)**

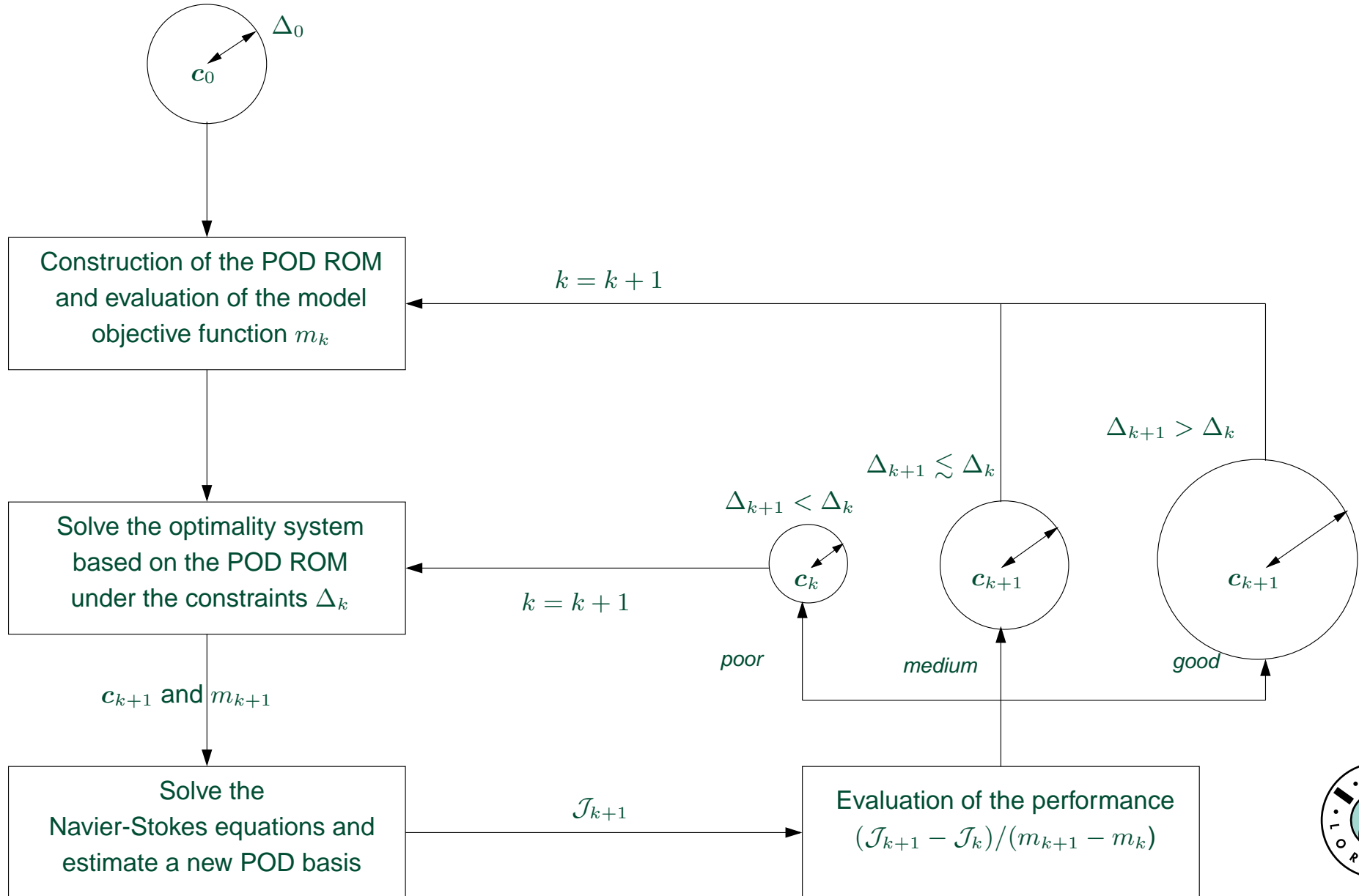
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Conn, A.R., Gould, N.I.M. et Toint, P.L. (2000) : Trust-region methods. *SIAM, Philadelphia.*



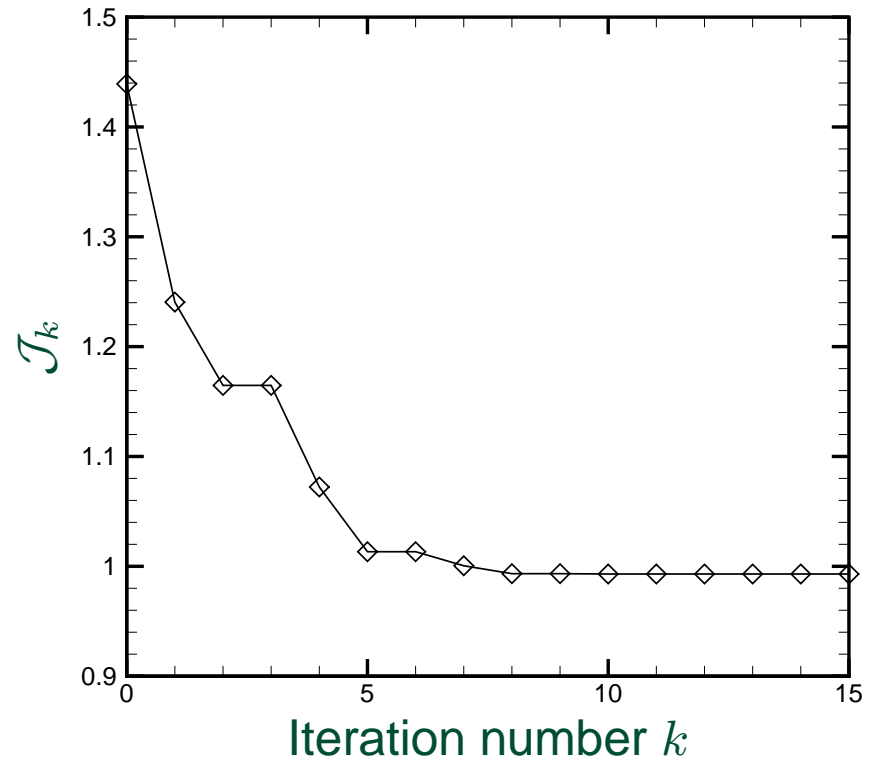
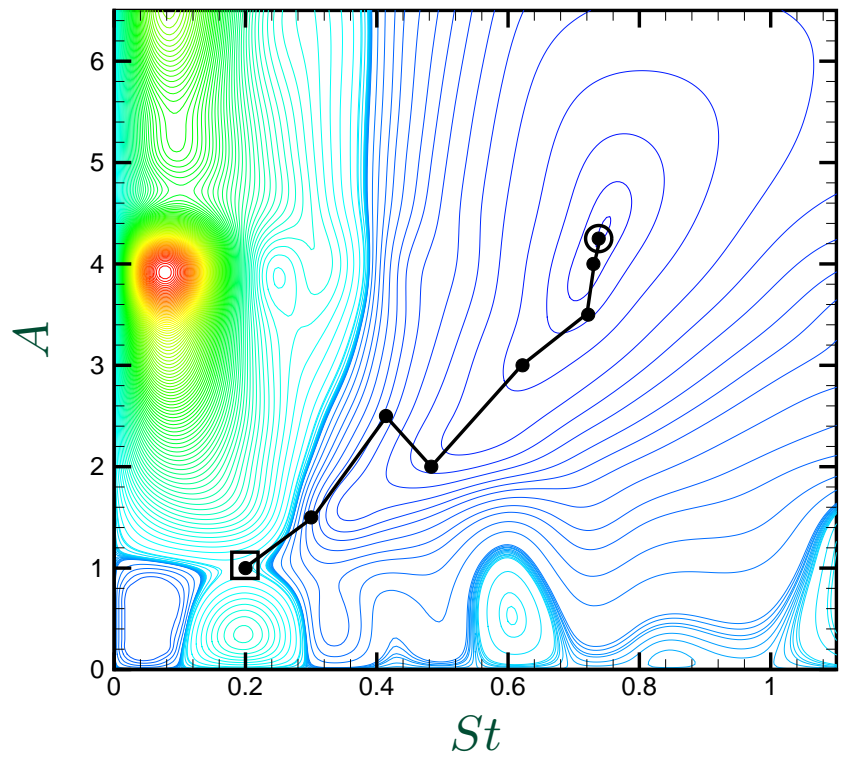
# Trust-Region Proper Orthogonal Decomposition (TRPOD) Algorithm

Initialization :  $c_0$ , Navier-Stokes resolution,  $\mathcal{J}_0$ .  $k = 0$ .





Initial control parameters :  $A = 1.0$  et  $St = 0.2$



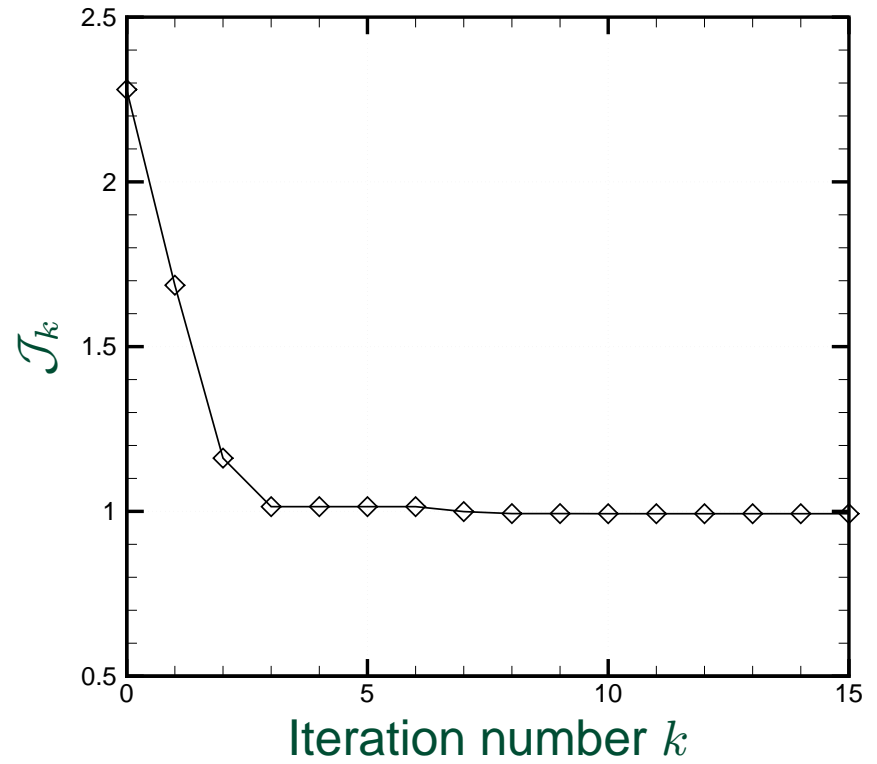
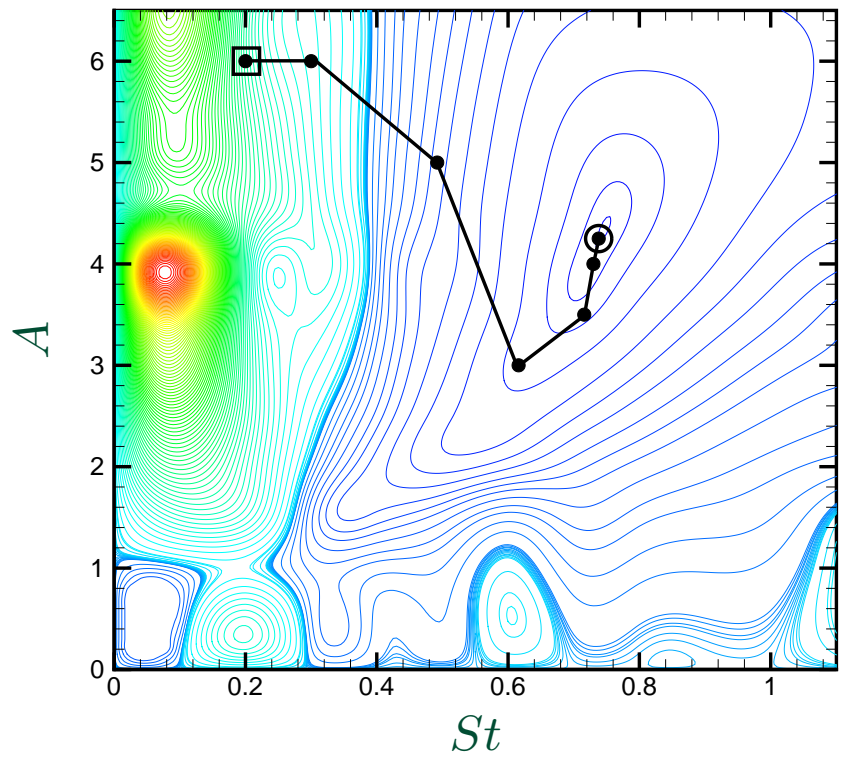
Optimal control parameters :  $A = 4.25$  et  $St = 0.74$

Mean drag coefficient :  $\mathcal{J} = 0.993$

8 resolutions of the Navier-Stokes equations



Initial control parameters :  $A = 6.0$  et  $St = 0.2$



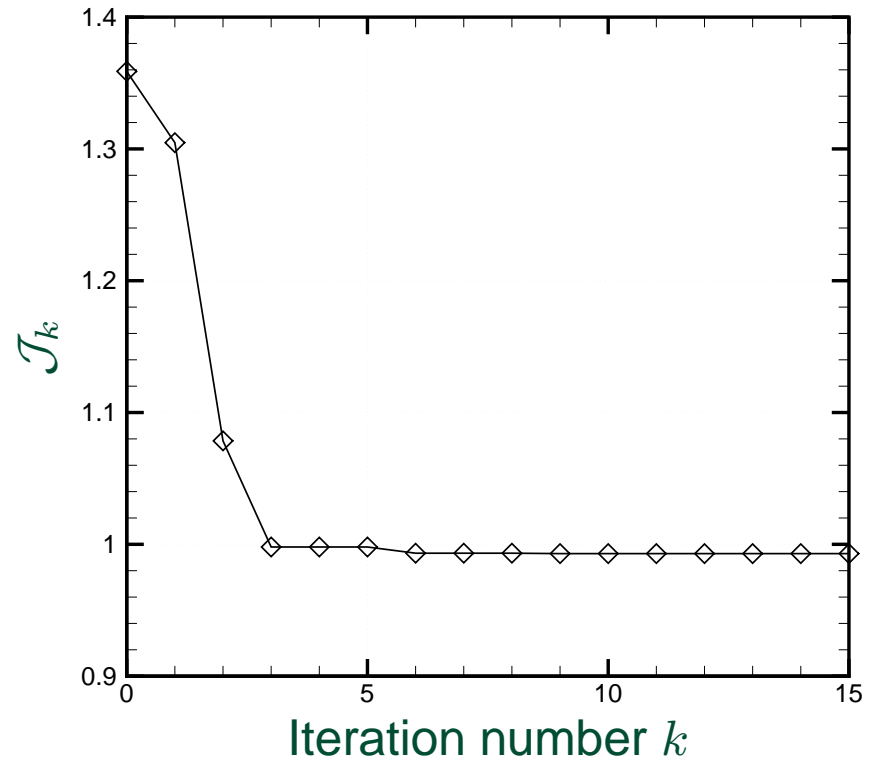
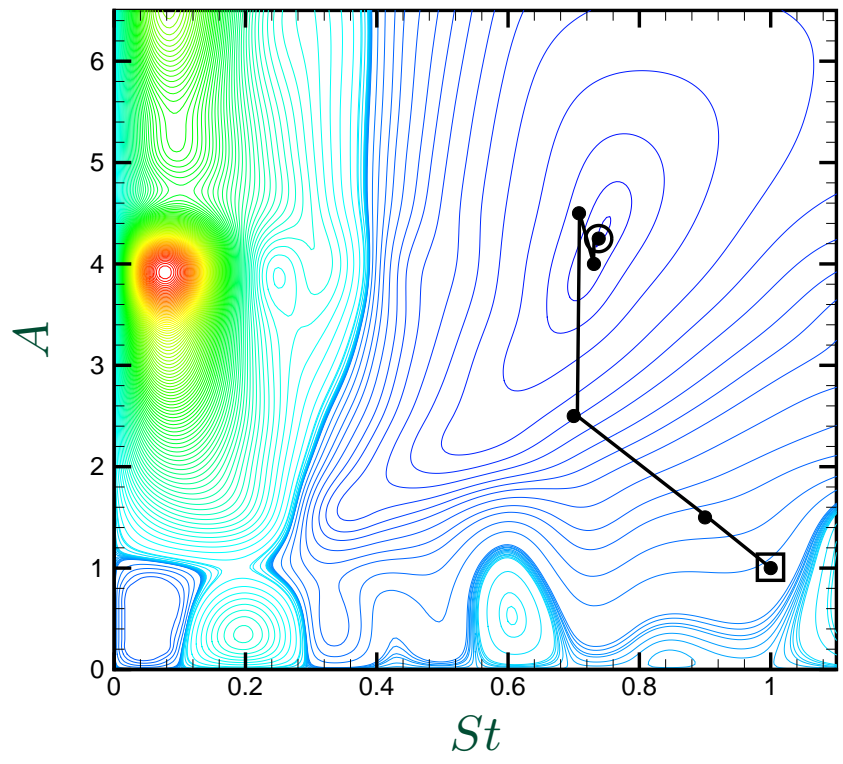
Optimal control parameters :  $A = 4.25$  and  $St = 0.74$

Mean drag coefficient :  $\mathcal{J} = 0.993$

6 resolutions of the Navier-Stokes equations



Initial control parameters :  $A = 1.0$  et  $St = 1.0$



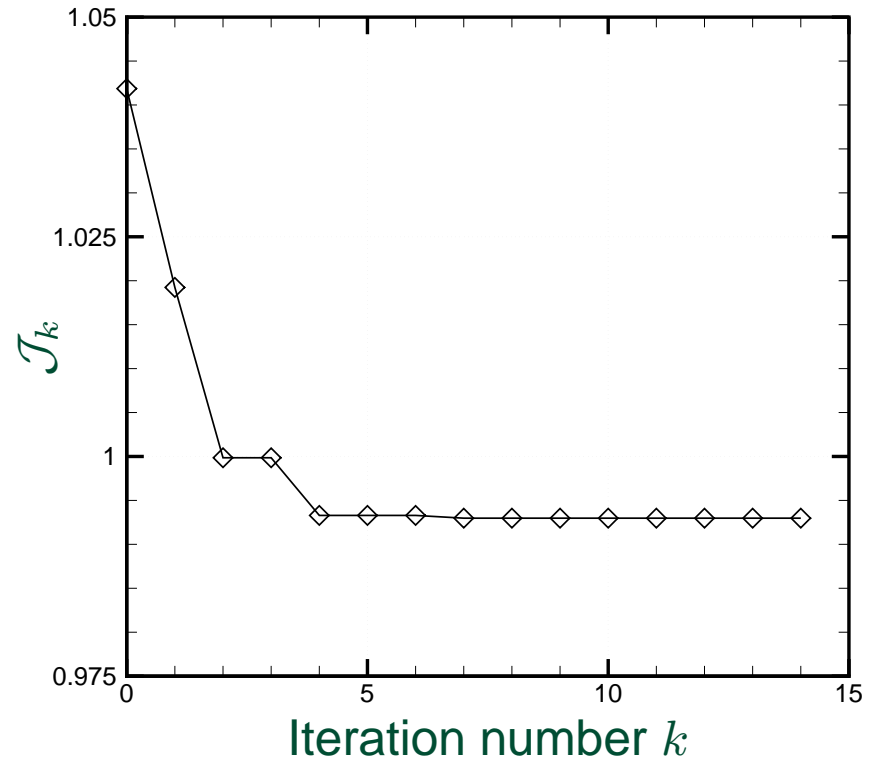
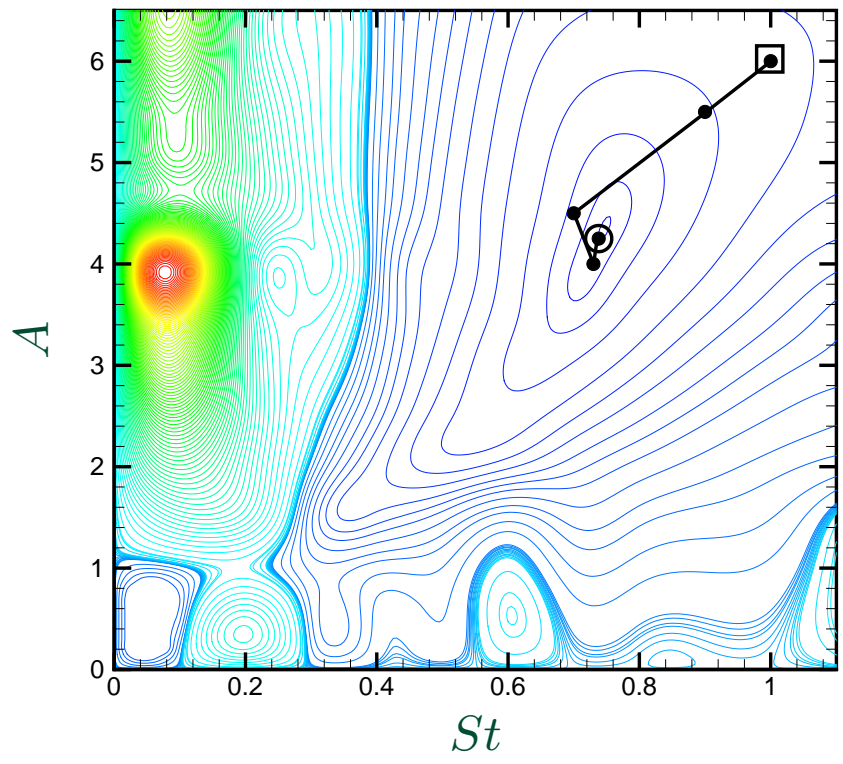
Optimal control parameters :  $A = 4.25$  and  $St = 0.74$

Mean drag coefficient :  $\mathcal{J} = 0.993$

5 resolutions of the Navier-Stokes equations



Initial control parameters :  $A = 6.0$  et  $St = 1.0$



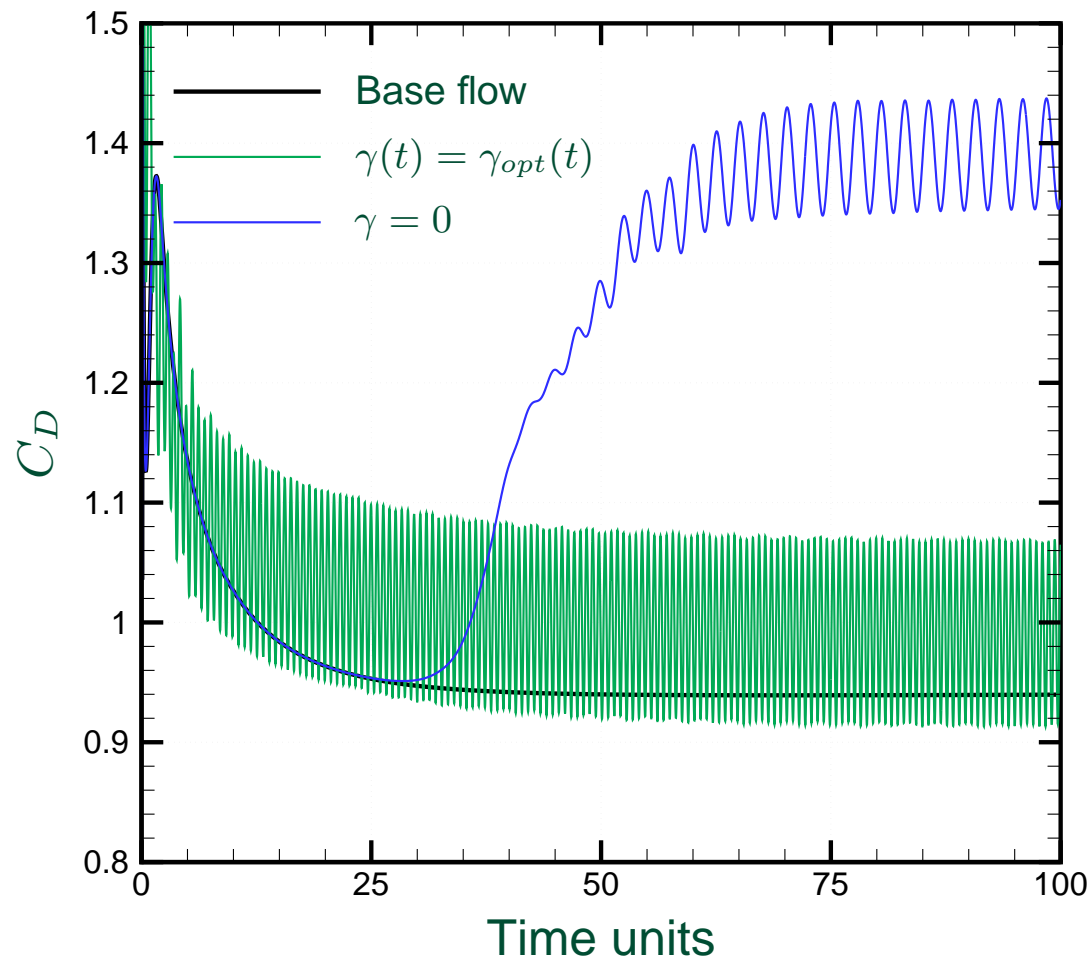
Optimal control parameters :  $A = 4.25$  and  $St = 0.74$

Mean drag coefficient :  $\mathcal{J} = 0.993$

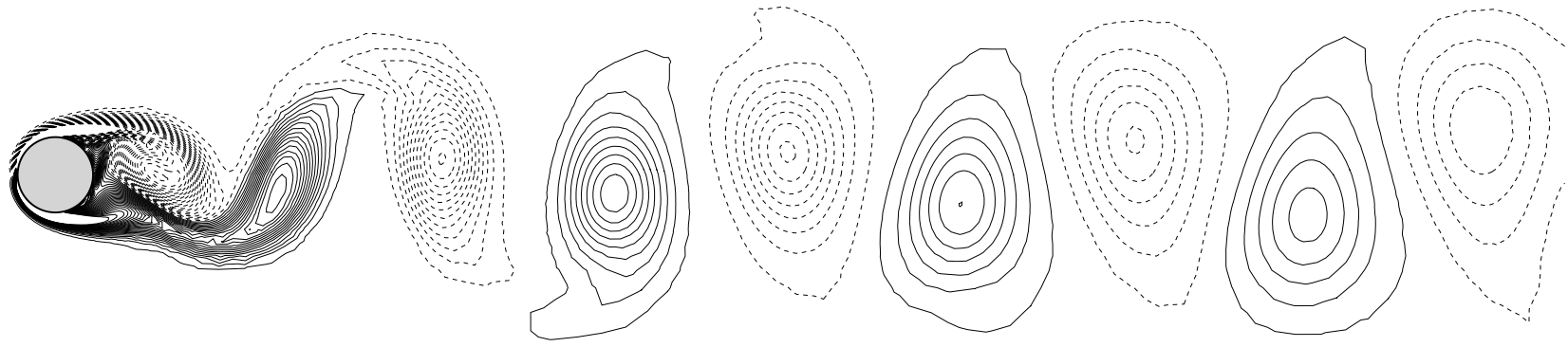
4 resolutions of the Navier-Stokes equations



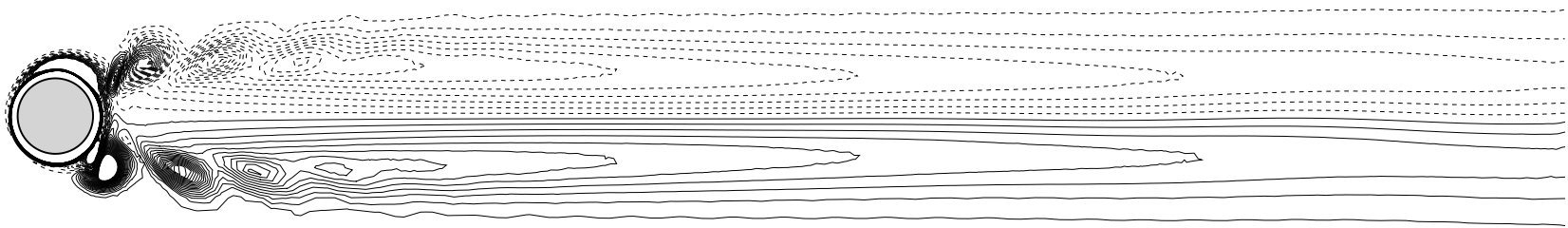
- Optimal control law :  $\gamma_{opt}(t) = A \sin(2\pi St t)$  avec  $A = 4.25$  et  $St = 0.74$



- Relative drag reduction of 30% ( $\mathcal{J}_0 = 1,4 \Rightarrow \mathcal{J}_{opt} = 0,99$ )



*Uncontrolled flow,  $\gamma = 0$ .*



*Controlled flow,  $\gamma = \gamma_{opt}$ .*

**Fig. :** *Iso-values of vorticity  $\omega_z$ .*

**Controlled flow : near wake strongly unsteady, far wake (after 5 diameters) steady and symmetric → steady unstable base flow**



- Optimal control of NSE by He *et al.* (2000) :
  - ⇒ 30% drag reduction for  $A = 3$  and  $S_t = 0.75$ .
- Optimal control POD ROM by Bergmann *et al.* (2005) with no reactualization of the POD ROM :
  - ⇒ 25% drag reduction for  $A = 2.2$  and  $S_t = 0.53$ .
  - Reduction costs compared to NSE :
    - CPU time : 100
    - Memory storage : 600but no mathematical proof concerning the Navier-Stokes optimality.
- TRPOD (this study) :
  - ⇒ More than 30% of drag reduction for  $A = 4.25$  and  $S_t = 0.738$ .
  - Reduction costs compared to NSE :
    - CPU time : 4
    - Memory storage : 400but global convergence.

↪ "Optimal" control of 3D flows becomes possible !



# Conclusions and perspectives

## ● Conclusions on TRPOD

- Important relative drag reduction : more than 30% of relative drag reduction
- **Global convergence** : mathematical assurance that the solution is identical to the one of the high-fidelity model
- TRPOD compared to NSE  $\Rightarrow$  important reduction of numerical costs :
  - $\hookrightarrow$  Reduction factor of the CPU time : 4
  - $\hookrightarrow$  Reduction factor of the memory storage : 400

**"OPTIMAL" CONTROL OF 3D FLOWS POSSIBLE BY POD ROM**

## ● Perspectives

- Optimal control of the channel flow at  $Re_\tau = 180$
- Test other reduced basis method than classical POD
  - Centroidal Voronoi Tessellations (Gunzburguer, 2004) : "intelligent" sampling in the control parameter space
  - Balanced POD (Rowley, 2004)
  - Model-based POD (Willcox, CDC-ECC 2005) : modify the definitions of the POD modes

