Optimal rotary control of the cylinder wake using POD Reduced Order Model

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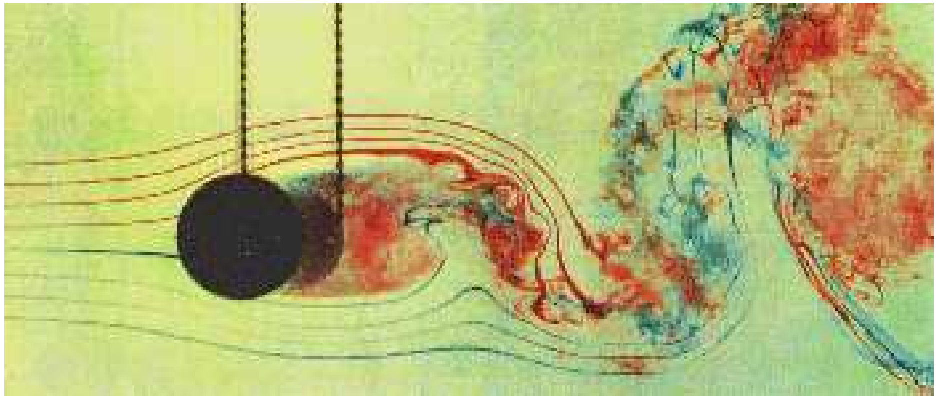
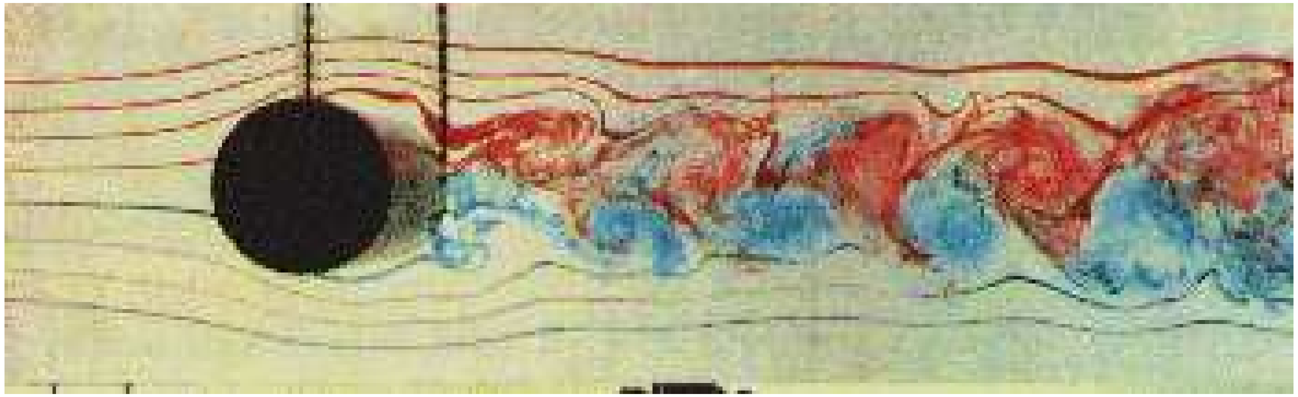
Outline

Motivations *Cylinder wake flow ?*

- Prototype configuration of separated flow
- Experimental study of Tokumaru and Dimotakis (JFM 1991)$Re = 15000$
  - ▶ Unforced flow



  - ▶ Forced flow $\Longrightarrow$ 80% of drag reduction

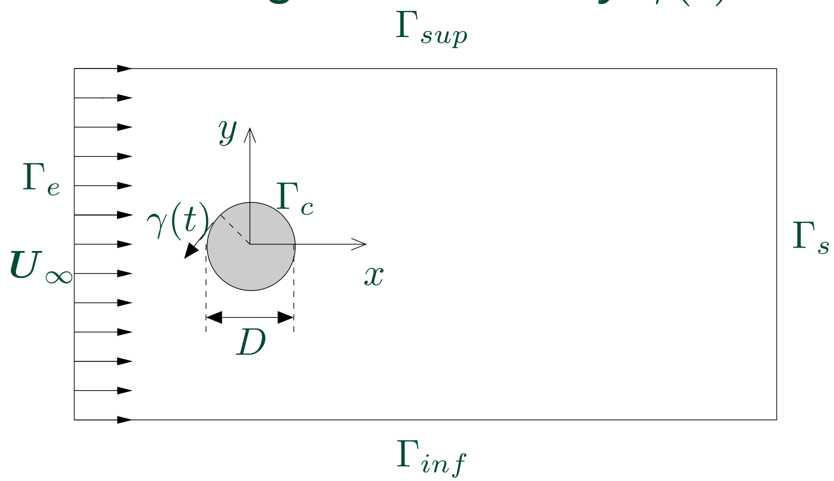


I - Configuration and numerical method

- Two dimensional flow around a circular cylinder at $Re = 200$
- Viscous, incompressible and Newtonian fluid

$$\boldsymbol{\nabla} \cdot \boldsymbol{u} = 0 \quad ; \quad \frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u} \cdot \boldsymbol{\nabla})\, \boldsymbol{u} = -\boldsymbol{\nabla} P + \frac{1}{Re} \boldsymbol{\Delta}\, \boldsymbol{u}$$

- Cylinder oscillation with a tangential velocity $\gamma(t)$

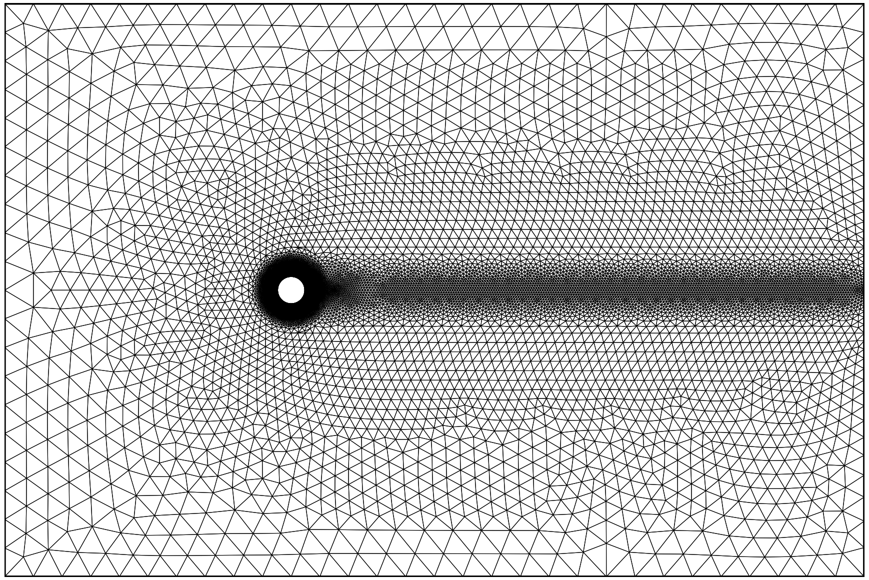


- Control parameter :

$$\alpha(t) = \frac{\gamma(t)}{U_\infty} = \frac{R\dot{\theta}(t)}{U_\infty} = \frac{\text{Tangential velocity}}{\text{Upstream velocity}}$$

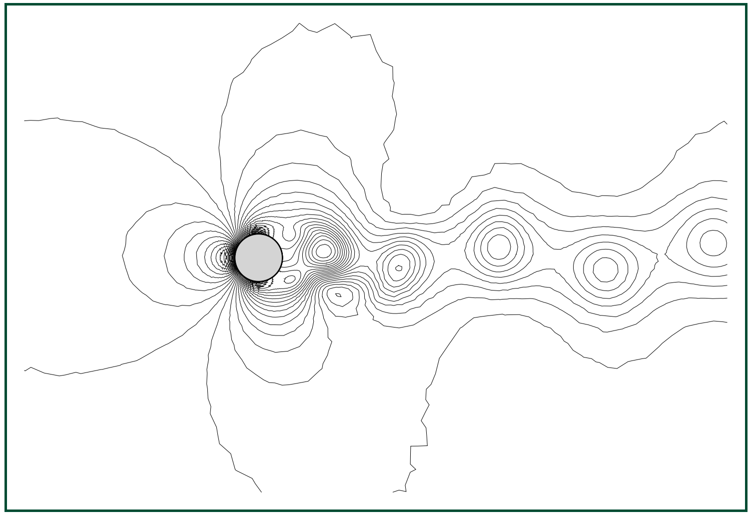
I - Configuration and numerical method

- Fractional step method in time (pressure correction)
- Finite Element Method (FEM) in space (P_1, P_1)
  - Numerical domain $\Omega = \{-10 \leq x \leq 20\,;\ -10 \leq y \leq 10\}$; $D = 1$
  - Mesh : 25042 triangles, 12686 vertices

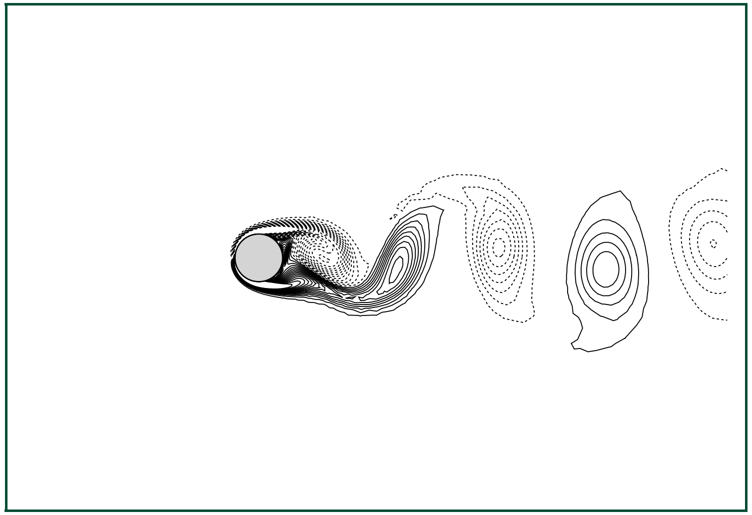


► *Numerical code written by M.Braza (IMFT-EMT2) & D.Ruiz (ENSEEIHT)*

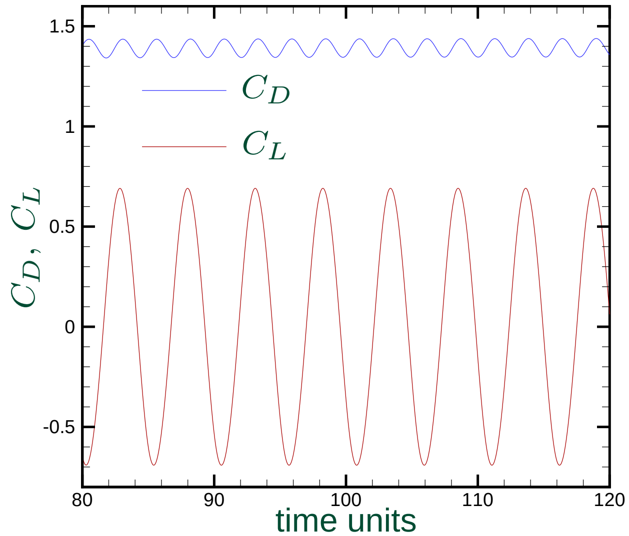
I - Configuration and numerical method



Iso pressure at $t = 100$.



Iso vorticity at $t = 100$.



Aerodynamic coefficients.

Authors	S_t	C_D
Braza *et al.* (1986)	0.2000	1.4000
Henderson *et al.* (1997)	0.1971	1.3412
He *et al.* (2000)	0.1978	1.3560
this study	**0.1983**	**1.3972**

Strouhal number and drag coefficient.

II - Optimal control *Definition*

Mathematical method allowing to determine without a priori knowledge a control law based on the optimization of a cost functional.

- State equations $\mathcal{F}(\phi, c) = 0$;
  (Navier-Stokes + I.C. + B.C.)
- Control variables c ;
  (Blowing/suction, design parameters ...)
- Cost functional $\mathcal{J}(\phi, c)$.
  (Drag, lift, target function, ...)

Find a control law c and state variables ϕ such that the cost functional $\mathcal{J}(\phi, c)$ reach an extremum under the constraint $\mathcal{F}(\phi, c) = 0$.

II - Optimal control *Lagrange multipliers*

Constrained optimization $\Rightarrow$ unconstrained optimization

- Introduction of Lagrange multipliers ξ (adjoint variables).
- Lagrange functional :

$$\mathcal{L}(\phi, c, \xi) = \mathcal{J}(\phi, c) - < \mathcal{F}(\phi, c), \xi >$$

- Force $\mathcal{L}$ to be stationary $\Rightarrow$ look for $\delta \mathcal{L} = 0$:

$$\delta \mathcal{L} = \frac{\partial \mathcal{L}}{\partial \phi} \delta \phi + \frac{\partial \mathcal{L}}{\partial c} \delta c + \frac{\partial \mathcal{L}}{\partial \xi} \delta \xi = 0$$

- Hypothesis : ϕ, c and ξ assumed to be independent of each other :

$$\frac{\partial \mathcal{L}}{\partial \phi} \delta \phi = \frac{\partial \mathcal{L}}{\partial c} \delta c = \frac{\partial \mathcal{L}}{\partial \xi} \delta \xi = 0$$

where

$$\frac{\partial \mathcal{L}}{\partial x} = \lim_{\epsilon \longrightarrow 0} \frac{\mathcal{L}(x + \epsilon \delta x) - \mathcal{L}(x)}{\epsilon} = 0 \quad \forall \delta x$$ (Fréchet derivative)

II - Optimal control *Optimality system*

- State equations $(\frac{\partial \mathcal{L}}{\partial \xi}\delta\xi = 0)$: $\boxed{\mathcal{F}(\phi, c) = 0}$
- Co-state (adjoint) equations $(\frac{\partial \mathcal{L}}{\partial \phi}\delta\phi = 0)$:

$$\boxed{\left(\frac{\partial \mathcal{F}}{\partial \phi}\right)^{*}\xi = \left(\frac{\partial \mathcal{J}}{\partial \phi}\right)^{*}}$$

- Optimality condition $(\frac{\partial \mathcal{L}}{\partial c}\delta c = 0)$:

$$\boxed{\left(\frac{\partial \mathcal{J}}{\partial c}\right)^{*} = \left(\frac{\partial \mathcal{F}}{\partial c}\right)^{*}\xi}$$

$\Rightarrow$ Expensive method in CPU time and storage memory for large system !

Bewley et al. (2000) : 10^8 grid points

$\Rightarrow$ Ensure only a local *(generally not global)* minimum

II - Optimal control *Iterative method*

- $c^{(0)}$ given ; for $n = 0, 1, 2,$ ... and while a convergence criterium is not satisfied, do :

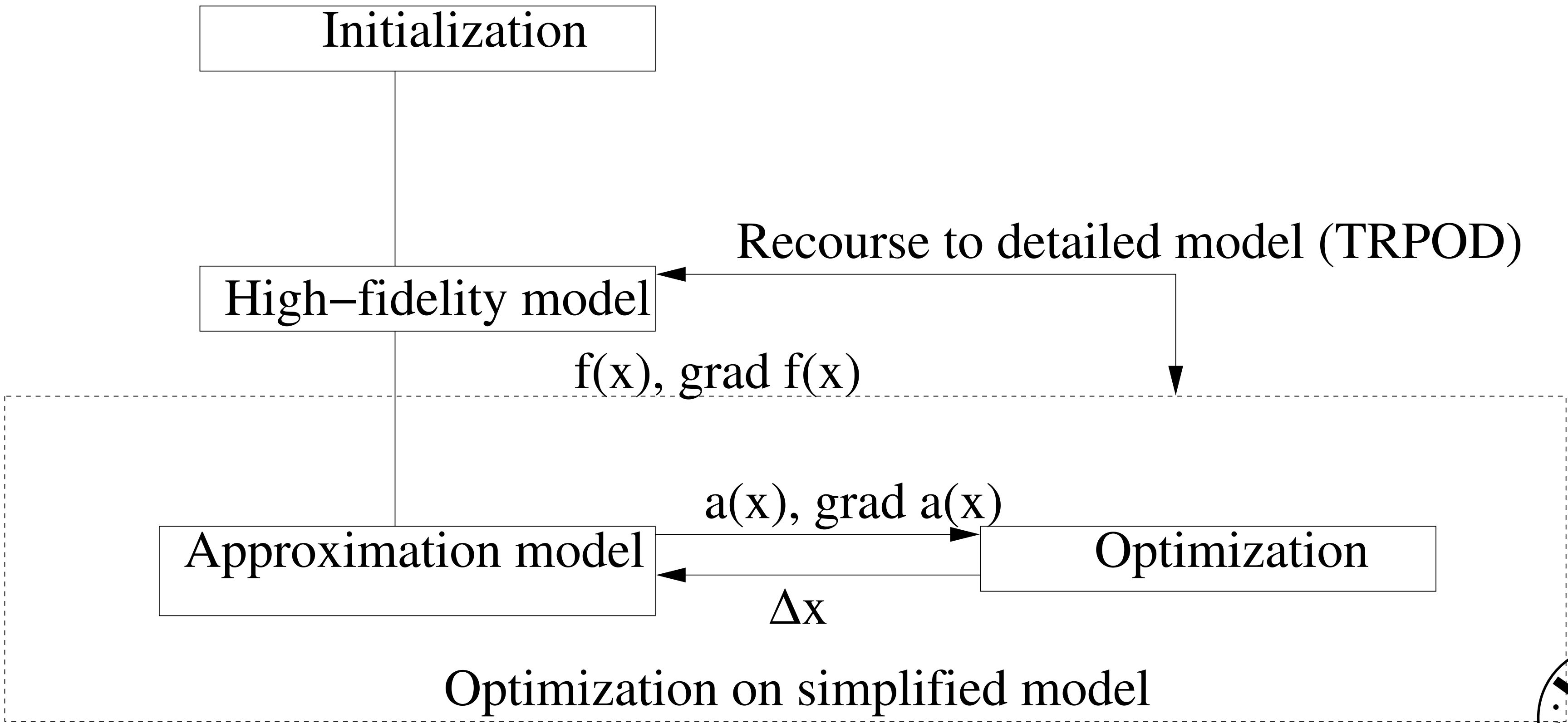
  1. From $t = 0$ to $t = T$ solve the state equations with $c^{(n)}$;
     $\hookrightarrow$ *state variables* $\phi^{(n)}$
  2. From $t = T$ to $t = 0$ solve the co-state equations with $\phi^{(n)}$;
     $\hookrightarrow$ *co-state variables* $\xi^{(n)}$
  3. Solve the optimality condition with $\phi^{(n)}$ and $\xi^{(n)}$;
     $\hookrightarrow$ *objective gradient* $\delta c^{(n)}$
  4. New control law $\hookrightarrow c^{(n+1)} = c^{(n)} + \omega^{(n)}\,\delta c^{(n)}$

- End do.

II - Optimal control *Reduced Order Model (ROM)*

"without an inexpensive method for reducing the cost of flow computation, it is unlikely that the solution of optimization problems involving the three dimensional unsteady Navier-Stokes system will become routine"

M. Gunzburger, 2000



III - Proper Orthogonal Decomposition (POD)

- ▶ Introduced in fluid mechanics (turbulence context) by Lumley (1967).

- ▶ Look for a realization $\phi(\boldsymbol{X})$ which is closer, in an average sense, to the realizations $\boldsymbol{u}(\boldsymbol{X})$. ($\boldsymbol{X} = (\boldsymbol{x}, t) \in \mathcal{D} = \Omega \times \mathbb{R}^+$)

- ▶ $\phi(\boldsymbol{X})$ solution of the problem : $\max_{\boldsymbol{\phi}} \langle |(\boldsymbol{u}, \boldsymbol{\phi}|^2 \rangle$ s.t. $\|\boldsymbol{\phi}\|^2 = 1.$

- ▶ Snapshots method, Sirovich (1987) :

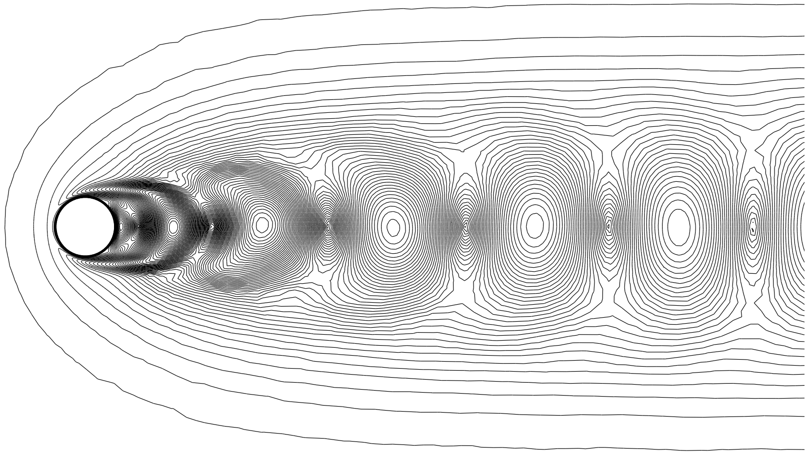
$$\int_T C(t, t') a^{(n)}(t') \, dt' = \lambda^{(n)} a^{(n)}(t).$$

- ▶ Optimal convergence L^2 *norm* (energy) of $\phi(\boldsymbol{X})$
  $\Rightarrow$ Dynamical order reduction is possible.

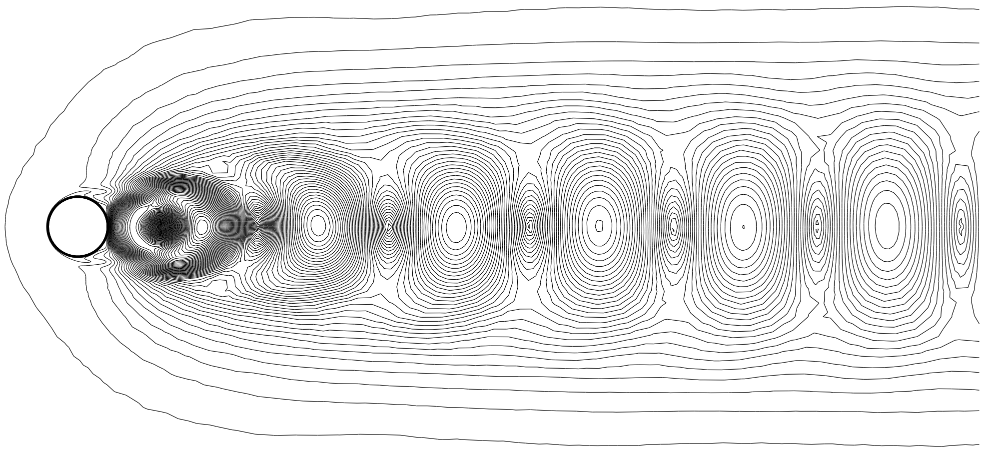
- ▶ Decomposition of the velocity field :

$$\boldsymbol{u}(\boldsymbol{x}, t) = \sum_{i=1}^{N_{POD}} a^{(i)}(t) \boldsymbol{\phi}^{(i)}(\boldsymbol{x}).$$

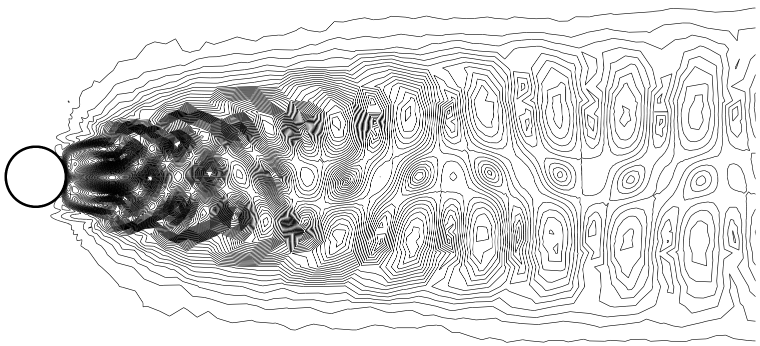
III - POD *POD modes : uncontrolled flow ($\gamma = 0$)*



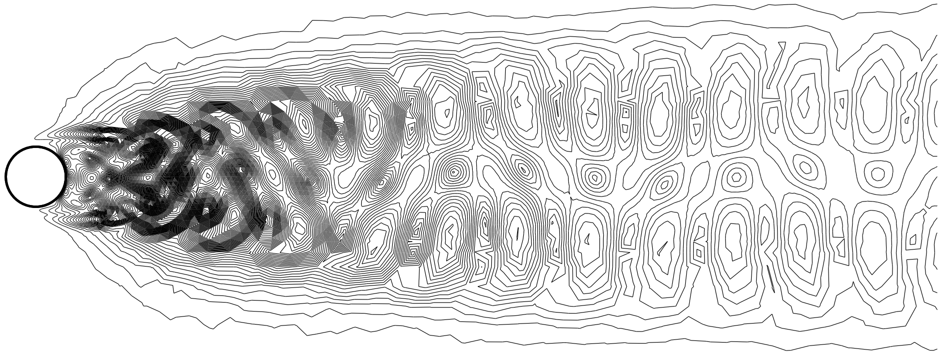
First POD mode.



Second POD mode.



Third POD mode.



Fourth POD mode.

III - Reduced Order Model of the cylinder wake (ROM)

- Galerkin projection of *NSE* on the POD basis :

$$\left(\boldsymbol{\phi}^{(i)},\ \frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u}\cdot\boldsymbol{\nabla})\boldsymbol{u}\right) = \left(\boldsymbol{\phi}^{(i)},\ -\boldsymbol{\nabla} p + \frac{1}{Re}\Delta \boldsymbol{u}\right).$$

- Integration by parts (Green's formula) leads :

$$\left(\boldsymbol{\phi}^{(i)},\ \frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u}\cdot\boldsymbol{\nabla})\boldsymbol{u}\right) = \left(p,\ \boldsymbol{\nabla}\cdot\boldsymbol{\phi}^{(i)}\right) - \frac{1}{Re}\left((\boldsymbol{\nabla}\otimes\boldsymbol{\phi}^{(i)})^T,\ \boldsymbol{\nabla}\otimes\boldsymbol{u}\right) - [p\,\boldsymbol{\phi}^{(i)}] + \frac{1}{Re}[(\boldsymbol{\nabla}\otimes\boldsymbol{u})\boldsymbol{\phi}^{(i)}].$$

with $[\boldsymbol{a}] = \int_\Gamma \boldsymbol{a}\cdot\boldsymbol{n}\,d\Gamma$ and $(A,\ B) = \int_\Omega A : B\,d\Omega = \sum_{i,\,j}\int_\Omega A_{ij}B_{ji}\,d\Omega$.

III - Reduced Order Model of the cylinder wake (ROM)

▶ Velocity decomposition with N_{POD} modes :

$$\boldsymbol{u}(\boldsymbol{x},t) = \boldsymbol{u}_m(\boldsymbol{x}) + \gamma(t)\,\boldsymbol{u}_c(\boldsymbol{x}) + \sum_{k=1}^{N_{POD}} a^{(k)}(t)\boldsymbol{\phi}^{(k)}(\boldsymbol{x}).$$

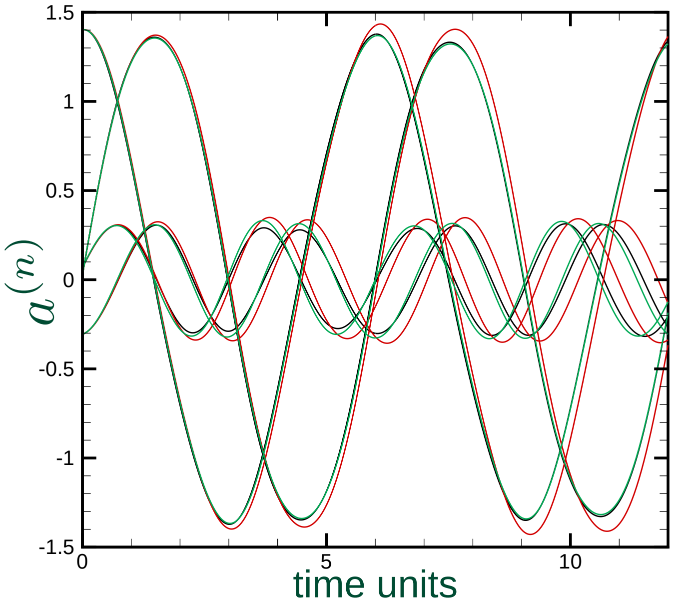
▶ Reduced order dynamical system where only N_{gal} $(\ll N_{POD})$ modes are retained (state equations) :

$$\begin{aligned}\frac{d\,a^{(i)}(t)}{d\,t} =& \mathcal{A}_i + \sum_{j=1}^{N_{gal}} \mathcal{B}_{ij}\,a^{(j)}(t) + \sum_{j=1}^{N_{gal}}\sum_{k=1}^{N_{gal}} \mathcal{C}_{ijk}\,a^{(j)}(t)a^{(k)}(t) \\ & + \mathcal{D}_i\,\frac{d\,\gamma}{d\,t} + \left(\mathcal{E}_i + \sum_{j=1}^{N_{gal}} \mathcal{F}_{ij}\,a^{(j)}(t)\right)\gamma + \mathcal{G}_i\gamma^2\end{aligned}$$

$$a^{(i)}(0) = (\boldsymbol{u}(\boldsymbol{x},\,0),\,\boldsymbol{\phi}^{(i)}(\boldsymbol{x})).$$

$\mathcal{A}_i$, $\mathcal{B}_{ij}$, $\mathcal{C}_{ijk}$, $\mathcal{D}_i$, $\mathcal{E}_i$, $\mathcal{F}_{ij}$ and $\mathcal{G}_i$ depend on $\boldsymbol{\phi}$, $\boldsymbol{u}_m$, $\boldsymbol{u}_c$ and Re.

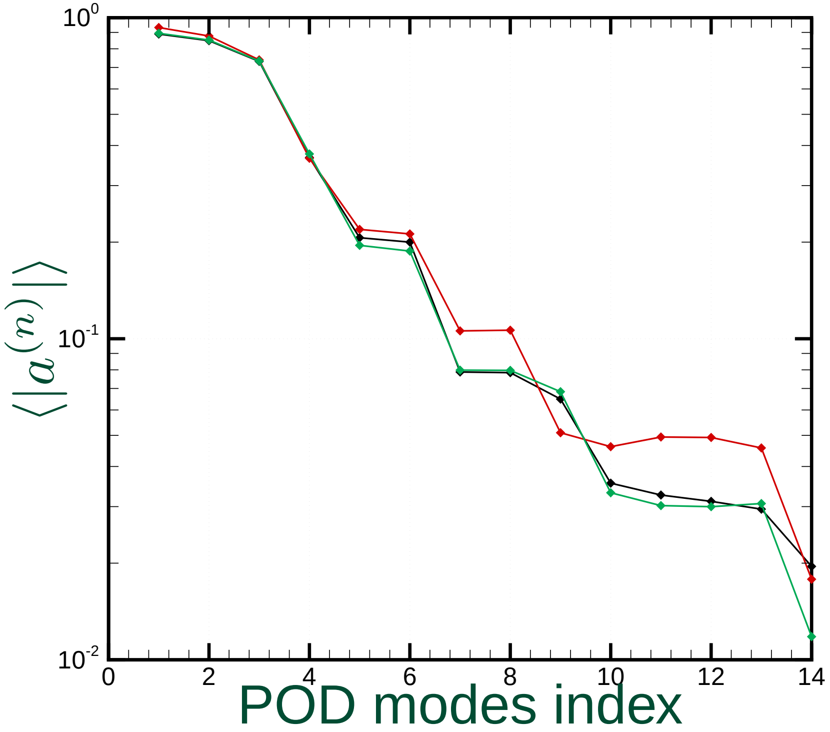
Integration and "optimal" stabilization of the POD ROM for
$\gamma = A \sin(2\pi S_t t)$, $A = 2$ and $S_t = 0.5$.
POD reconstruction errors $\Rightarrow$ temporal modes amplification



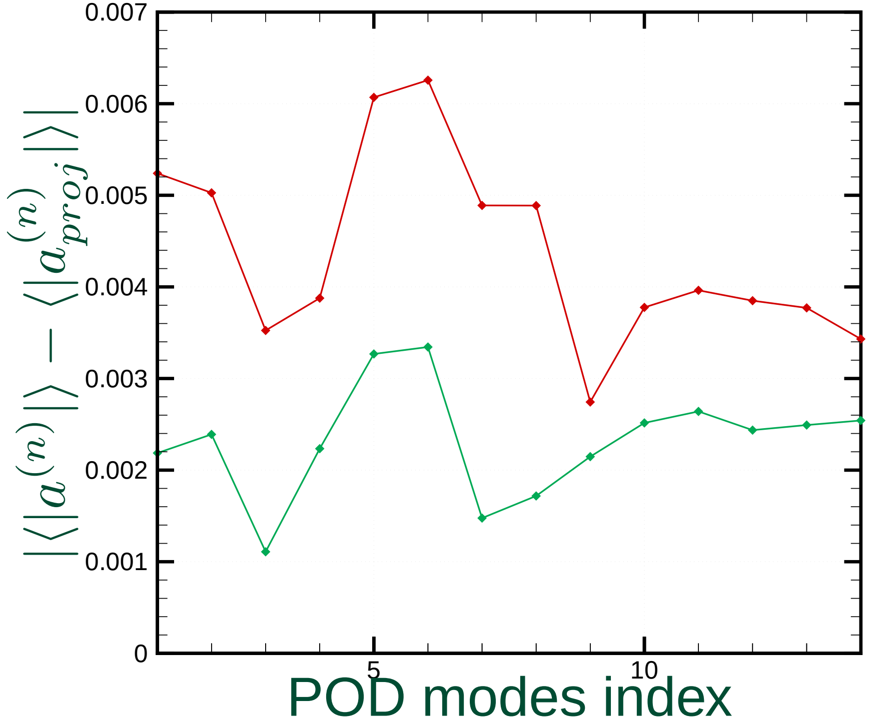
Temporal evolution of the first 6 POD temporal modes.

- Reasons :
  - Extraction by POD only of the large energetic eddies
  - Dissipation takes place in small eddies
- Solution :
  - Addition of an optimal artificial viscosity on each POD mode

projection (Navier-Stokes)
prediction before stabilization (POD ROM)
prediction after stabilization (POD ROM).



Comparison of energetic spectrum.



Comparison of absolute errors.

- ▶ Good agreements between POD ROM spectrum and DNS spectrum
- ▶ Reduction of the reconstruction error between predicted (POD ROM) and projected (DNS) modes

$\Rightarrow$ Validation of the POD ROM

V - Optimal control formulation based on ROM

- Objective functional :

$$\mathcal{J}(\boldsymbol{a}, \gamma(t)) = \int_0^T J(\boldsymbol{a}, \gamma(t))\,dt = \int_0^T \left(\sum_{i=1}^{N_{gal}} {a^{(i)}}^2 + \frac{\alpha}{2}\gamma(t)^2 \right) dt.$$

α : regularization parameter (penalization).

- Co-state equations :

$$\frac{d\,\xi^{(i)}(t)}{dt} = -\sum_{j=1}^{N_{gal}} \left(\mathcal{B}_{ji} + \gamma\,\mathcal{F}_{ji} + \sum_{k=1}^{N_{gal}} \left(\mathcal{C}_{jik} + \mathcal{C}_{jki}\right) a^{(k)} \right) \xi^{(j)}(t) - 2a^{(i)}$$

$$\xi^{(i)}(T) = 0.$$

- Optimality condition (gradient) :

$$\delta\gamma(t) = -\sum_{i=1}^{N_{gal}} \mathcal{D}_i \frac{d\xi^{(i)}}{dt} + \sum_{i=1}^{N_{gal}} \left(\mathcal{E}_i + \sum_{j=1}^{N_{gal}} \mathcal{F}_{ij} a^{(j)} + 2\mathcal{G}_i\gamma(t) \right) \xi^{(i)} + \alpha\gamma.$$

VI - Results of POD ROM *Generalities*

► No reactualization of the POD basis.

► The energetic representativity is *a priori* different to the dynamical one :

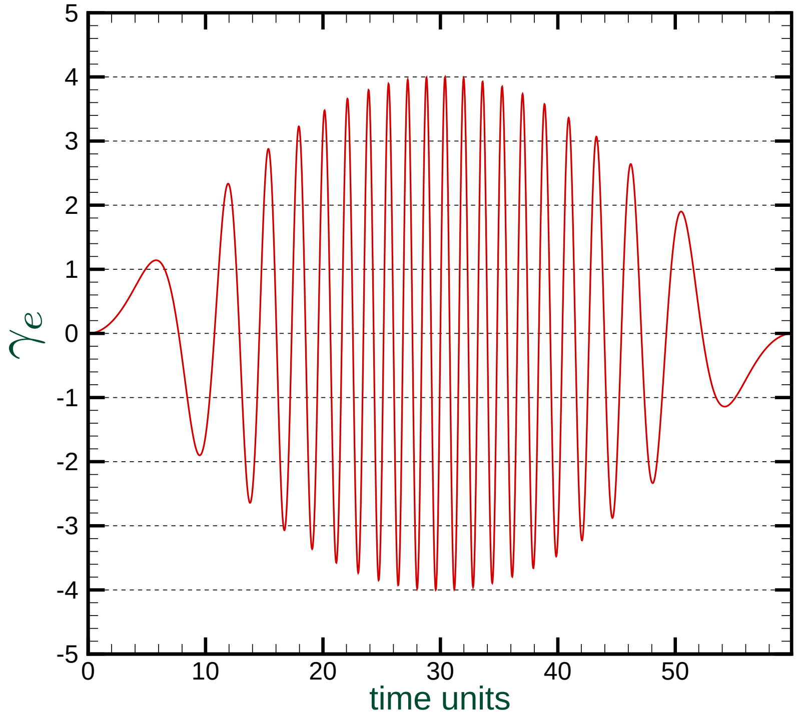
$\hookrightarrow$ possible inconvenient for control,

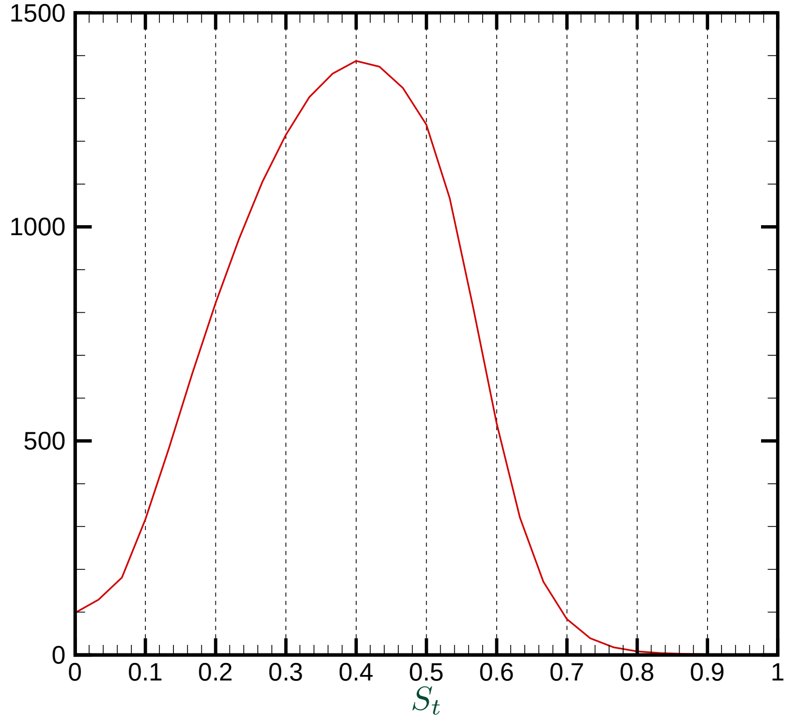
$\hookrightarrow$ a POD dynamical system represents *a priori* only the dynamics (and its vicinity) used to build the low dynamical model.

► Construction of a POD basis representative of a large range of dynamics :

$\hookrightarrow$*excitation of a great number of degrees of freedom scanning* $\gamma(t)$ *in amplitudes and frequencies.*

VI - Results of POD ROM *Excitation*



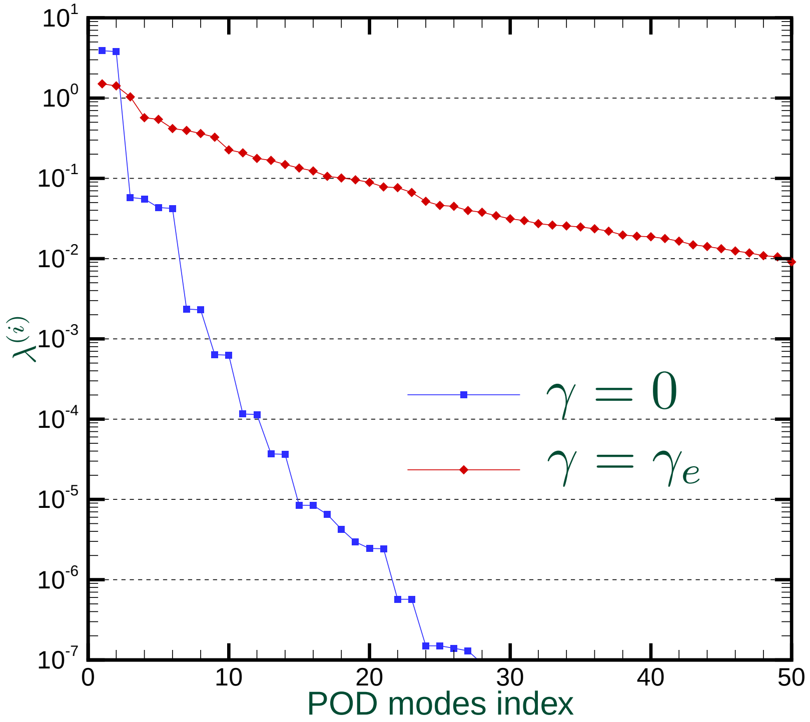


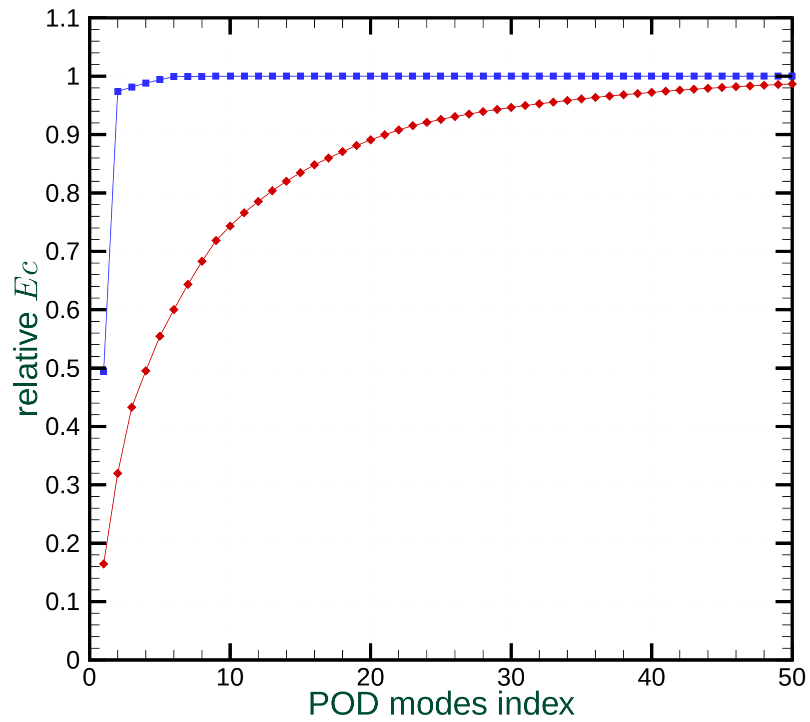
$$\gamma_e(t) = A_1 \sin(2\pi S_{t1}\, t) \times \sin(2\pi S_{t2}\, t - A_2 \sin(2\pi S_{t3}\, t))$$

with $A_1 = 4$, $A_2 = 18$, $S_{t1} = 1/120$, $S_{t2} = 1/3$ and $S_{t3} = 1/60$.

- ► $0 \leq$ amplitudes ≤ 4 and Fourier analysis $\Rightarrow 0 \leq$ frequencies ≤ 0.65
- ► γ_e initial control law in the iterative process.

VI - Results of POD ROM *Energy*



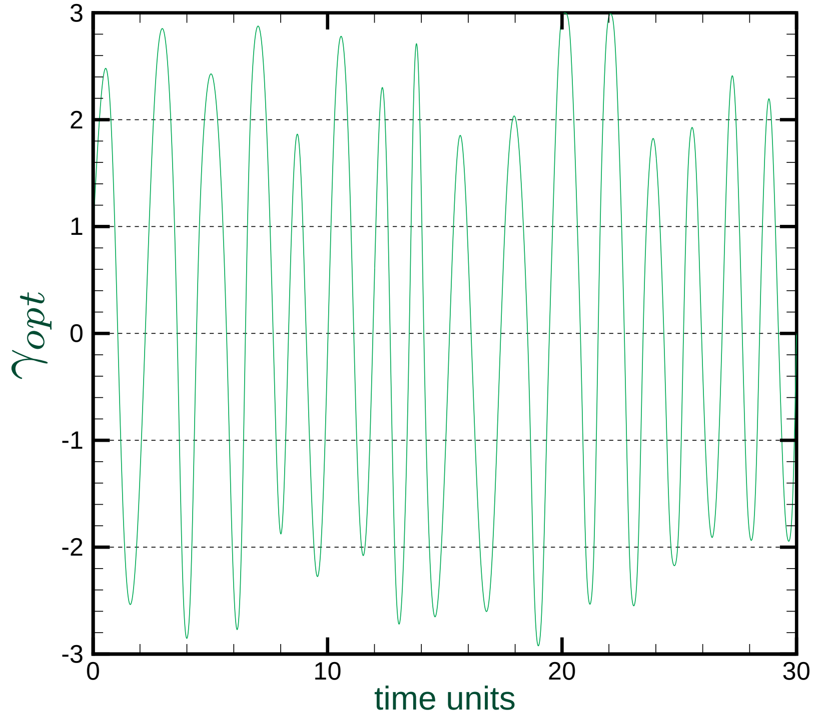


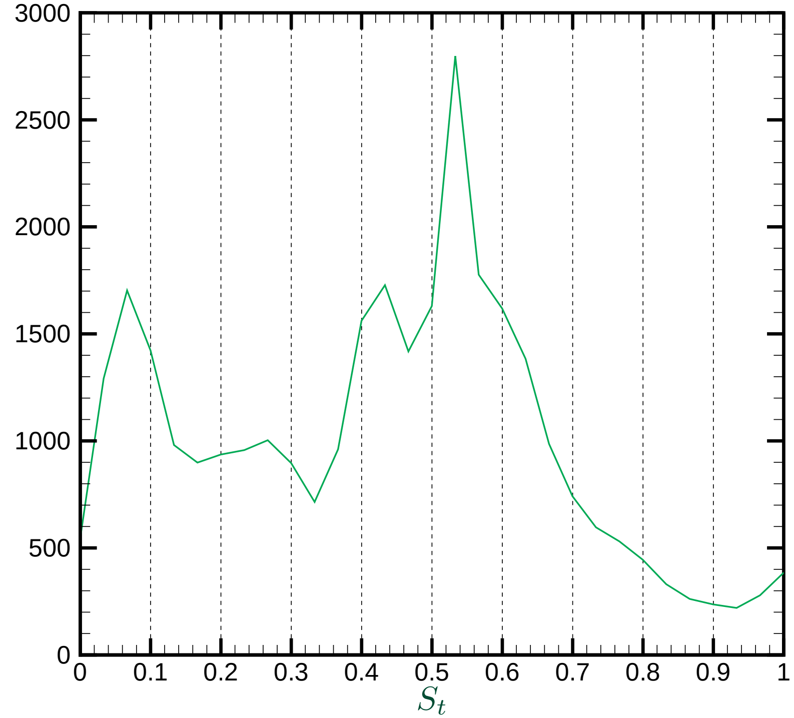
▶ Stationary cylinder $\gamma = 0$: $\hookrightarrow 2$ *modes out of 100 are sufficient to restore* 97% *of the kinetic energy.*

▶ Controlled cylinder $\gamma = \gamma_e$: $\hookrightarrow 40$ *modes out of 100 are then necessary to restore* 97% *of the kinetic energy*

$\Rightarrow$ *Improvement of the POD ROM robustness to dynamical evolutions.*

VI - Results of POD ROM *Optimal control*





▶ Reduction of the wake instationarity. $\gamma_{opt} \simeq A \sin(2\pi S_t t)$ with $A = 2.2$ and $S_t = 0.53$

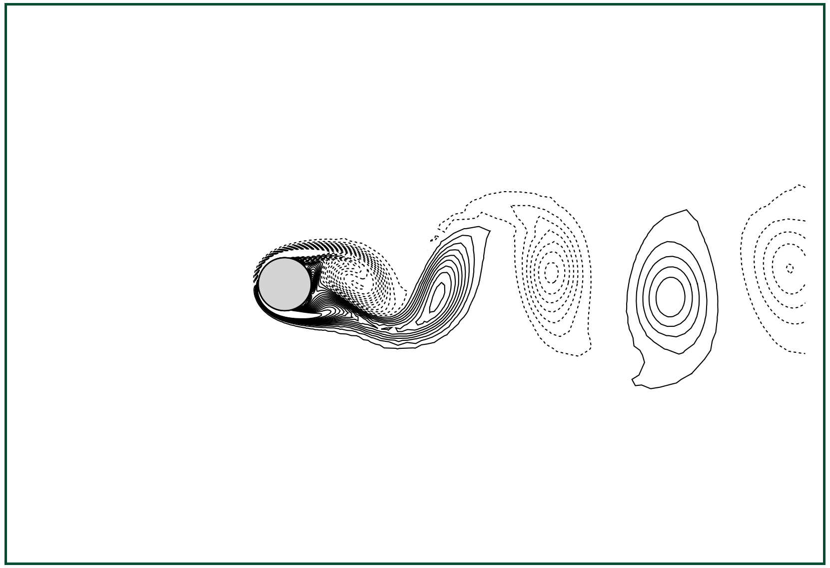
$$\mathcal{J}(\gamma_e) = 9.81 \quad \Longrightarrow \quad \mathcal{J}(\gamma_{opt}) = 5.63.$$

▶ The control is optimal for the reduced order model based on POD.

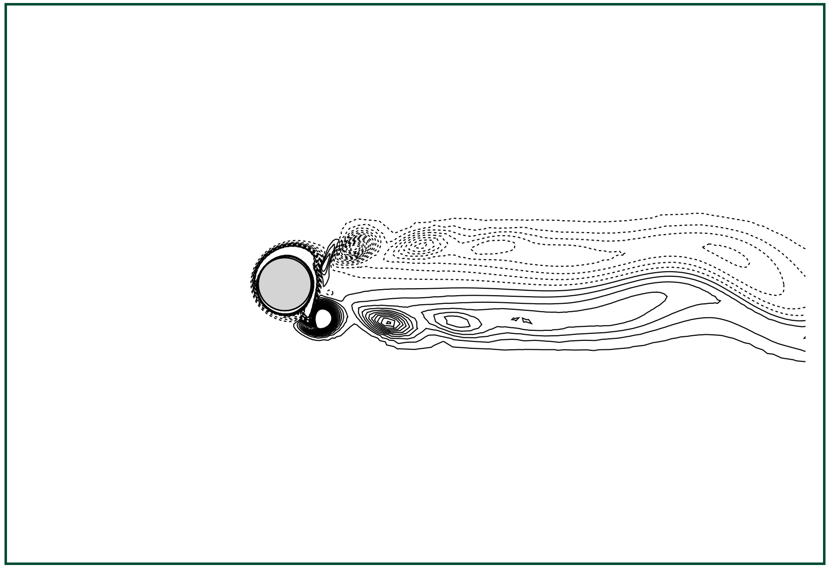
▶ Is it also optimal for the Navier-Stokes model ?

VI - Results of POD ROM *Comparison of wakes' organization*

- No mathematical proof concerning the Navier Stokes optimality.



no control $\gamma = 0$

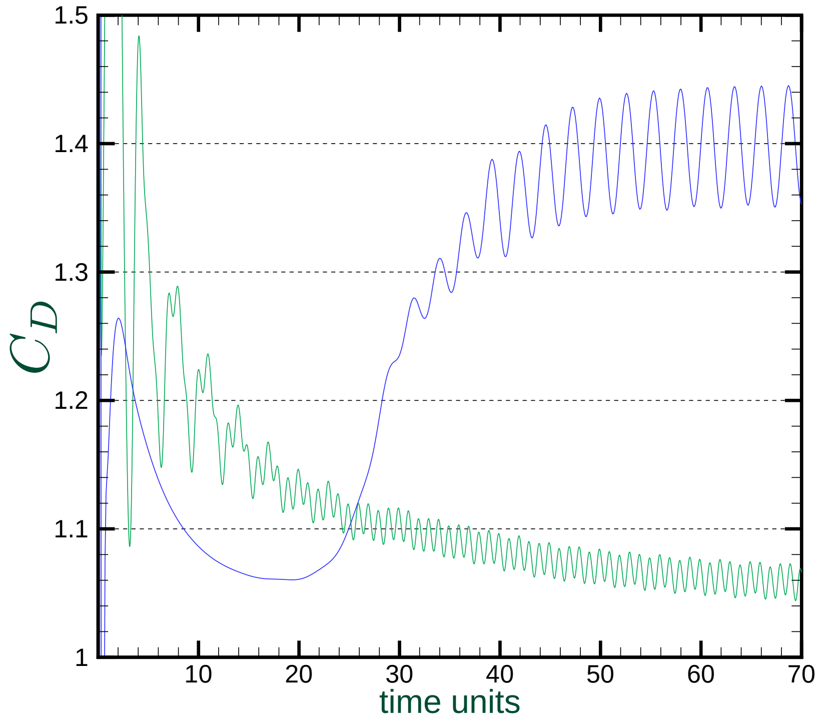


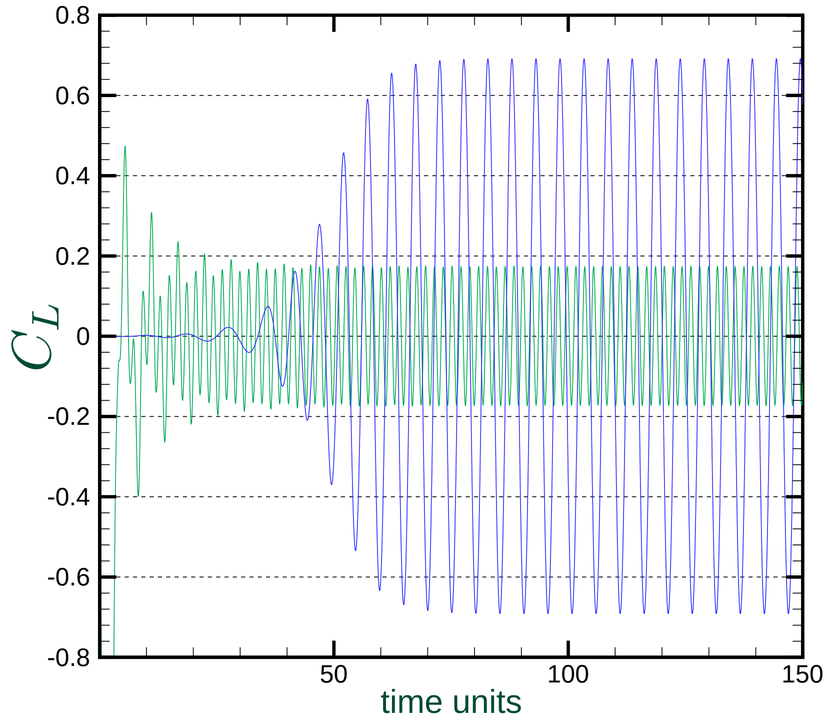
optimal control $\gamma = \gamma_{opt}$

Isocontours of vorticity ω_z.

- no control : $\gamma = 0 \Rightarrow$ Asymmetric flow.
  $\hookrightarrow$ Large and energetic eddies.
- optimal control : $\gamma = \gamma_{opt} \Rightarrow$ Symmetrization of the (near) wake.
  $\hookrightarrow$ Smaller and lower energetic eddies.

VI - Results of POD ROM *Aerodynamic coefficients*





- Important drag reduction :

$$C_{D0} = 1.40 \text{ for } \gamma = 0 \text{ and } C_D = 1.04 \text{ for } \gamma = \gamma_{opt}$$
$$C_D/C_{D0} = 0.74 \Rightarrow \text{more than } 25\%.$$

- Decrease of the lift amplitude :

$$C_L = 0.68 \text{ for } \gamma = 0 \text{ and } C_L = 0.13 \text{ for } \gamma = \gamma_{opt}.$$

VI - Results of POD ROM *Numerical costs*

▶ Optimal control of NSE by He *et al.* (2000) :

$\hookrightarrow$ harmonic control law with $A = 3$ and $S_t = 0.75$.

$\Rightarrow 30\%$ drag reduction.

▶ Optimal control POD ROM (this study) :

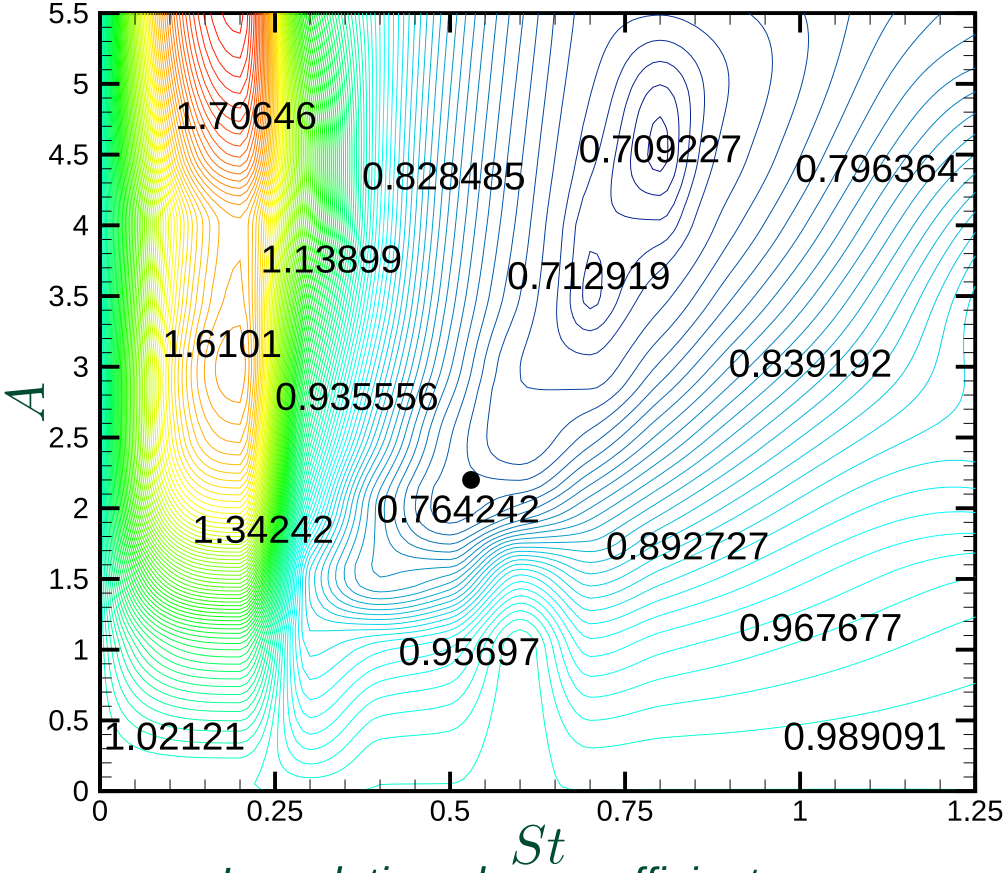
$\hookrightarrow$ harmonic control law with $A = 2.2$ and $S_t = 0.53$.

$\Rightarrow 25\%$ drag reduction.

- Reduction costs using POD ROM compared to NSE :
  - CPU time : 100
  - Memory storage : 600

$\hookrightarrow$ "Optimal" control of 3D flows becomes possible !

▶ Does the POD ROM control law correspond to the global minimum ?



Iso-relative- drag coefficient
$C_D(A, St)/C_{D0}$ *in* (A, St) *plan.*

Observations

- ▶ Minimum is located in a smooth valley

$\hookrightarrow$ *Global minimum :*
around $A = 4.4$ *and* $St = 0.76$

- ▶ Maximum is located in a sharp peak

$\hookrightarrow$ *Global maximum :*
near $St = 0.2$*, the natural frequency : lock-on flow*

Finding the global minimum with an optimization algorithm may be difficult due to the smooth valley

VI - Results of POD ROM *Local versus global minimum*

- POD ROM control law does not correspond to the global minimum

  $\hookrightarrow$ POD ROM parameters : $A = 2.2$ and $St = 0.53 \Rightarrow C_D = 1.04$

  $\hookrightarrow$ Global minimum parameters : $A = 4.4$ and $St = 0.76 \Rightarrow C_D = 0.98$

- Results in (A, St) quite different but not so far in terms of C_D

  $\hookrightarrow$ The smooth valley is reached

- Improvement : coupling to the POD ROM approach an efficient new optimization algorithm for smooth fonctions

  $\hookrightarrow$ Take results obtained by POD ROM as initial conditions

VII - Nelder-Mead Simplex method *Generalities*

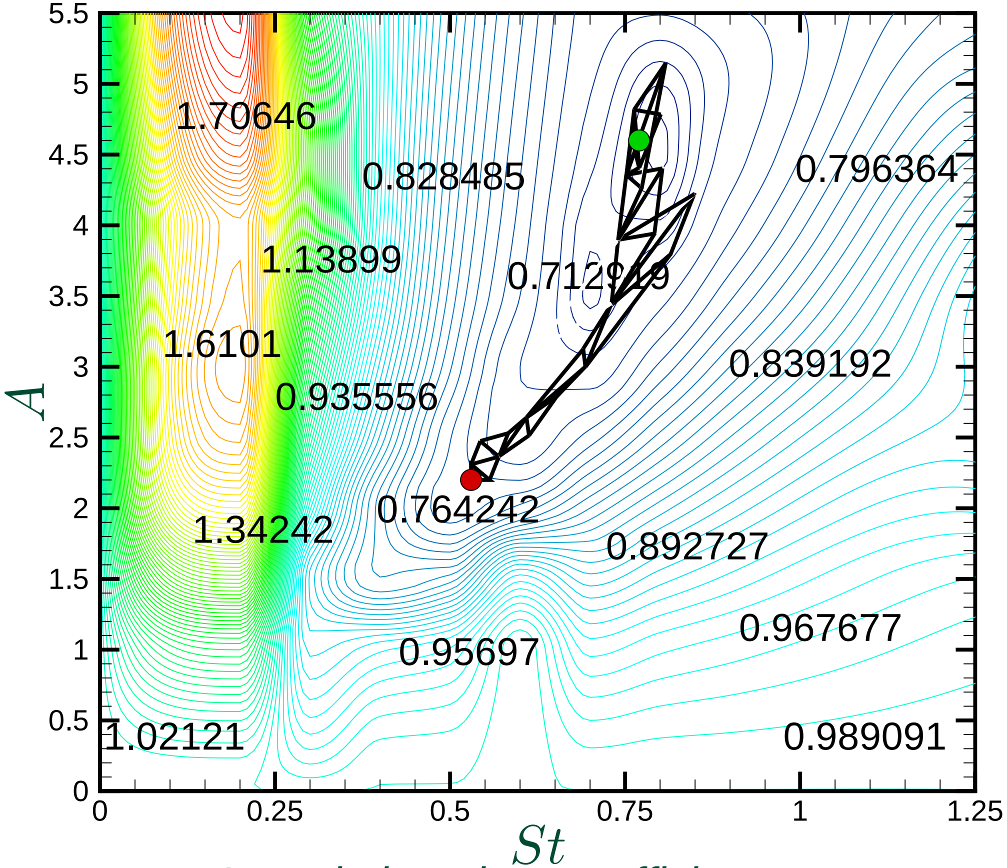
Advantages

- Numerical simplicities
- Adaptive topology
- Free gradient optimization method
- Good results with smooth functions

Drawbacks

- No proof of optimality for simplex dimensions greater than two
- Need to fix free parameters
- Maybe more iterations than gradient based optimisation algorithms...

VII - Nelder-Mead Simplex method *Results*



Iso-relative- drag coefficient
$C_D(A,\, St)/C_{D0}$ *in* (A, St) *plan.*

Observations

- ▶ Topology adaptation function of the curve of the valley
- ▶ Minimum found by the simplex method :

$$A = 4.5 \text{ and } St = 0.76$$

$\hookrightarrow$ Seems to be the global minimum

- ▶ 30 NSE resolutions $\Rightarrow$ 5% additive drag reduction compared to POD ROM

Relative drag reduction by POD ROM : 25% (1 NSE resolution)
Usefulness of coupling a new algorithm ?

Conclusions

- Important drag reduction obtained by POD ROM : more than 25% of relative drag reduction
- This solution is not the global minimum of the drag function
- POD ROM compared to NSE $\Rightarrow$ important reduction of numerical costs :
  $\hookrightarrow$ Reduction factor of the CPU time : 100
  $\hookrightarrow$ Reduction factor of the memory storage : 600

  "OPTIMAL" CONTROL OF 3D FLOWS POSSIBLE BY POD ROM

- Coupling POD ROM with the Nelder-Mead simplex method leads *a priori* to the global minimum of the drag function
- But the gain on the drag function is quite small (5%) compared to results obtained by POD ROM

Perspectives

- Improve the representativity of the POD ROM

$\hookrightarrow$ "Optimize" the temporal excitation γ_e
$\hookrightarrow$ Mix snapshots corresponding to different dynamics (temporal excitations)
$\hookrightarrow$ Introduction of shift-mode ?

- Look for harmonic control $\gamma(t) = A \sin(2\pi\, S_t\, t)$ with POD basis reactualization (closed loop on NSE and not only on POD ROM)

- Coupling the POD ROM approach with Trust Region POD method (TRPOD)

$\Longrightarrow$ proof of convergence under weak conditions

- Introducing the pressure into the POD dynamical system

$\hookrightarrow$ pressure contribution to drag coefficient : 80%

- Optimal control of the Navier-Stokes equations