# WAVELET-BASED LEVEL SET EVOLUTION FOR CLASSIFICATION OF TEXTURED IMAGES

*Jean-François Aujol*[1,2] *, *Gilles Aubert*[1] *and Laure Blanc-Féraud*[2]

[1] Labo J.A. Dieudonné, Université de Nice, France
[2] Projet Ariana (Cnrs/Inria/Unsa), Inria Sophia-Antipolis, France

## ABSTRACT

We present a supervised classification model based on a variational approach. This model is specifically devoted to textured images. We want to get a partition of an image, composed of texture regions separated by regular interfaces. Each kind of texture defines a class. We use a wavelet packet transform to analyze the textures, charactized by their energy distribution in each sub-band. In order to have an image segmentation according to the classes, we model the regions and their interfaces by level set functions. We define a functional on these level sets whose minimizers define the optimal classification according to textures. A system of coupled PDEs is deduced from the functional. By solving this system, each region evolves according to its wavelet coefficients and interacts with the neighbour region in order to obtain a partition with regular contours. Experiments are shown on synthetic and real images.

## 1. INTRODUCTION

Image classification consists in assigning a label to each pixel of an observed image. This label indicates to which class belongs a pixel. Classification is one of the basic problem in image processing. It concerns many applications as for instance landscape management in teledetection. The classification problem is closely related to the segmentation one, in the sense that the goal is to get a partition of the image composed of homogeneous regions. In the classification problem, each partition is assigned to a class. Many classification models have been developed, especially from regions growing algorithms [1], or by a stochastic approach [2], and most recently by a variational approach [3, 4].

Our model is inspired by the work of C. Samson $et\ al.$ who have developed a supervised classification algorithm for non textured images in [3]. In this model, a complete segmentation of an image is obtained through level set evolution, where each level set represents the regions of a given class. The number of classes $K$ present in the image is supposed to be known, as well as the charactistics of each class. The goal is then to assign a class to each pixel of an image in order to define a classified image. Regularization is needed to avoid isolated pixels or too oscillatory boundaries.

The paper is organized as follows. Section 2 presents the framework of our classification algorithm. We then charactize textures through their wavelet expansion in section 3. We deduce our data term from this study in section 4, and we present the algorithm we use to minimize it. We then present some numerical results on both synthetic and real textured images in section 5. We refer the reader to [5] for a complete insight of our model.

## 2. CLASSIFICATION

### 2.1. Partition, level set approach

The observed image is considered as a function $u_0 : \Omega \mapsto \mathbb{R}$ (where $\Omega$ is an open subset of $\mathbb{R}^2$). We denote $\Omega_k = \{x/\, x \in Cl_k\}$, where $Cl_k$ is class number $k$, $1 \leq k \leq K$. The collection of open sets $\{\Omega_k\}$ must form a partition of $\Omega$. We denote $\Gamma_k = \partial\Omega_k \bigcap \Omega$ the boundary of $\Omega_k$ (except points belonging also to $\partial\Omega$).

In order to get a functional formulation rather than a set formulation, we assign to each $\Omega_k$ a lipschitz function $\Phi_k : \Omega \to \mathbb{R}$ such that:

$$\begin{cases} \Phi_k(x) > 0 & \text{if } x \in \Omega_k \\ \Phi_k(x) = 0 & \text{if } x \in \Gamma_k \\ \Phi_k(x) < 0 & \text{otherwise} \end{cases}$$

$\Omega_k$ is thus completely determined by $\Phi_k$.

To define the classification problem as a functional minimization, we use Dirac and Heaviside distributions $\delta$ and $H$, or more precisely classical regular approximations of these distributions.

$$\delta_\alpha(s) = \begin{cases} \frac{1}{2\alpha}\left(1 + \cos\frac{\pi s}{\alpha}\right) & \text{if } |s| \leq \alpha \\ 0 & \text{if } |s| < \alpha \end{cases}$$

$$H_\alpha(s) = \begin{cases} \frac{1}{2}\left(1 + \frac{s}{\alpha} + \frac{1}{\pi}\sin\frac{\pi s}{\alpha}\right) & \text{si } |s| \leq \alpha \\ 1 & \text{if } s > \alpha \\ 0 & \text{if } s < -\alpha \end{cases}$$

When $\alpha \to 0$, we have $\delta_\alpha \to \delta$ and $H_\alpha \to H$ (in the distributional sense).

### 2.2. Functional

Our functional is composed of three terms:

1.
$$F^A_\alpha(\Phi_1, \ldots, \Phi_K) = \lambda \int_\Omega \left( \sum_{k=1}^{K} H_\alpha(\Phi_i) - 1 \right)^2 \tag{1}$$

This energy term ensures that the result of the functional minimization is indeed a partition of the image support $\Omega$, then defines a complete segemntation of the observed image $u_0$.

2.
$$F^B(\Phi_1, \ldots, \Phi_K) = \gamma \sum_{k=1}^{K} |\Gamma_k| \tag{2}$$

This term penalizes the contour length, which prevent from having too irregulars contours. By using the co-area formula, it is possible to show that (the proof is given in [3]):

$$\lim_{\alpha \to 0} \int_\Omega \delta_\alpha(\Phi_k) |\nabla \Phi_k| dx = |\Gamma_k| \tag{3}$$

In practice, we seek to minimize:

$$F^B_\alpha(\Phi_1, \ldots, \Phi_K) = \gamma \sum_{k=1}^{K} \int_\Omega \delta_\alpha(\Phi_k) |\nabla \Phi_k| \tag{4}$$

3.
$$F^C(\Phi_1, \ldots, \Phi_K) \tag{5}$$

This last term stands for the data term, and we will get it from our texture modelization and the maximum likelihood principle (see section 3 below).

## 3. TEXTURE MODELIZATION

### 3.1. Idea

We consider that a texture is charactized by the energy of its wavelets coefficients. If we note $u_0$ the function which represents this texture, we can write:

$$u_0 = \sum_n u_{J,n} \phi_{J,n} + \sum_{j=-J}^{-1} \sum_n w_{j,n} \psi_{j,n} \tag{6}$$

where $\psi$ is the mother wavelet, $\phi$ the scaling function and $J$ the order of the decomposition (see [6]). As in [1], we consider that a texture is characterized by the sequence:

$$\left( (|u_{J,n}|^2, n \in \mathbb{Z}), (|w_{j,n}|^2, n \in \mathbb{Z}, -J \leq j \leq -1) \right) \tag{7}$$

### 3.2. Probability distribution

S.G. Mallat checked experimentally (see [6]) that the distribution of the modulus of the wavelets coefficients in a sub-band follows a generalized gaussian law of the form:

$$p_X(x) = A \exp\left( - \left( \frac{x}{\alpha} \right)^\beta \right) \mathbb{I}_{x \geq 0} \tag{8}$$

As we consider that textures are characterized by their energy, we compute the distribution law of the square of the wavelet coefficients in a sub-band.

$$p_{X^2}(y) = \frac{A}{2\sqrt{y}} \exp\left( - \left( \frac{\sqrt{y}}{\alpha} \right)^\beta \right) \mathbb{I}_{y \geq 0} \tag{9}$$

The parameters $\alpha$ and $\beta$ can be computed from the first and second order moments. We recall that the function $\Gamma$ is defined on $\mathbb{R}_+$ by $\Gamma(t) = \int_0^{+\infty} e^{-u} u^{t-1} du$. By using the fact that $\int_\mathbb{R} p_X(x) dx = 1$ , we get $A = \frac{\beta}{\alpha \Gamma(\frac{1}{\beta})}$. Let us denote $Y = X^2$. We compute the first and second order moments of $Y$, $M_1 = E(Y)$ and $M_2 = E(Y^2)$. We then deduce: $\alpha = \sqrt{\frac{\Gamma(\frac{3}{\beta})}{\Gamma(\frac{5}{\beta})} \frac{M_2}{M_1}}$ and $\beta = F^{-1}\left( \frac{M_1^2}{M_2} \right)$ with $F(x) = \frac{\Gamma^2(\frac{3}{x})}{\Gamma(\frac{1}{x})\Gamma(\frac{5}{x})}$.

## 4. COMPLETE FUNCTIONAL

### 4.1. Hypotheses

From the modelization we made in the previous section, we now deduce the data term we use in our algorithm. We compute a wavelet packet decomposition of the image (up to the second order in practice): we get $I$ channels ($I = 16$ in practice).

In what follows, we call the energy at pixel $s$ the vector $U(s) = (u_1(s), \ldots, u_I(s))$, where $u_i(s)$ is the square of the wavelet coefficient in the sub-band $i$ at pixel $s$. We also make the two following hypotheses:

**(H1)** We assume that, for each texture $k = 1 \ldots K$, in each channel $i = 1 \ldots I$, the square of the wavelet coefficients follows a law of the type (9) of mean $M_1^{k,i}$ and of second order moment $M_2^{k,i}$. This is obtained by a preprocessing.

**(H2)** We consider that the different channels are independent. This is an approximation but it allows simple modeling.

### 4.2. Data term

The goal is to find for each pixel the class which makes the observed energy $U$ the most likely. In other words, we use the *maximum likelihood estimator* for the data term. In order to write the energy in terms of probability, we discretize the set $\Omega$. We denote by $S$ the set of the pixels. From the observations $U$, the class $Cl$ is the one which maximizes the likelihood $P(U|Cl)$. According to hypotheses (H1) and (H2), the likelihood probability density is:

$$f(u|Cl=k)=\prod_{i=1}^{I}\frac{A_k^i}{2\sqrt{u_i}}\exp\left(-\left(\frac{\sqrt{u_i}}{\alpha_k^i}\right)^{\beta_k^i}\right) \quad (10)$$

The parameters $A_k^i$, $\alpha_k^i$ and $\beta_k^i$ are computed from the first and second order moments. In fact, we are going to maximize the log-likelihood, which amounts to minimize:

$$-\ln(P(U|Cl))=-\sum_{k=1}^{K}\sum_{s\in S}\ln(P(U_s|Cl_s=k))\delta_k(Cl_s) \quad (11)$$

where $\delta_k(s)=1$ if $s$ belongs to the class $k$, 0 otherwise. By using (10), it can be deduced:

$$-\ln(P(U|Cl))=-\sum_{k=1}^{K}\sum_{s\in S}\sum_{i=1}^{I}B_k^i(s)\delta_k(Cl_s) \quad (12)$$

where

$$B_k^i(s)=-\ln A_k^i+\ln 2+\frac{1}{2}\ln u_i(s)+\left(\frac{\sqrt{u_i(s)}}{\alpha_k^i}\right)^{\beta_k^i} \quad (13)$$

We rewrite now (12) in a continuous framework. We get:

$$\begin{aligned}-\ln(P(U|Cl)) &= \sum_{k=1}^{K}\sum_{i=1}^{I}\int_{\Omega_k}B_k^i(x)dx \quad (14)\\ &= F^C(\Phi_1,\dots,\Phi_K)\end{aligned}$$

Using $\Phi_k$ and $H_\alpha$, we have:

$$F_\alpha^C(\Phi_1,\dots,\Phi_K)=\sum_{k=1}^{K}\sum_{i=1}^{I}\int_\Omega H_\alpha(\Phi_k)B_k^i(x)dx \quad (15)$$

### 4.3. The functional

We are now able to completely write the functional which models the classification problem for textured images:

$$F(\Phi_1,\dots,\Phi_K)=\sum_{k=1}^{K}\sum_{i=1}^{I}\int_\Omega H_\alpha(\Phi_k)B_k^i(x)dx$$
$$+\frac{\lambda}{2}\int_\Omega\left(\sum_{k=1}^{K}H_\alpha(\Phi_k)-1\right)^2+\gamma\sum_{k=1}^{K}\int_\Omega\delta_\alpha(\Phi_k)|\nabla\Phi_k|$$

### 4.4. Dynamical scheme

Assuming that Neumann conditions are verified, the associated Euler-Lagrange equations to $F$ (see [7]) give a system composed of $K$-coupled PDE's. To solve this system, we embed it in the following dynamical scheme ($k=1\dots K$):

$$\begin{aligned}\frac{\partial\Phi_k}{\partial t}=-\delta_\alpha(\Phi_k)\Bigg[\lambda\left(\sum_{q=1}^{K}H_\alpha(\Phi_q)-1\right) \quad (16)\\ -\gamma\,\mathrm{div}\left(\frac{\nabla\Phi_k}{|\nabla\Phi_k|}\right)+\left(\sum_{i=1}^{I}B_k^i(x)\right)\Bigg]\end{aligned}$$

with as initial condition $\Phi_k(0,x)$ the Euclidean signed distance function to the zero level set of $\Phi_k$ (see section below for the initialization of these zero level sets). We discretize this system with finite differences (see [3]).

## 5. NUMERICAL RESULTS

### 5.1. Choice of parameters

**Wavelets:** As we deal with texture, we use undecimated wavelets so that our data term is translation invariant. We use a wavelet packet decomposition, because texture dominant frequencies also lie in median frequencies channels. As the mean of the gray level must not be a feature for a texture, we modify the low frequency block of the wavelet packet decomposition by setting its mean to zero. We have tested different kinds of wavelets, and we have chosen to use the Daubechies wavelet with ten vanishing moments (see [6]).

**Information given by the user:** As the classification is here supervised, the user has to give the number of classes (textures), as well as the parameters of each class (the first and second order moments of the energy distribution in each sub-band of the packet wavelet decomposition).

**Parameters:** The functional contains three terms, we have two parameters to tune. First, the partition parameter $\lambda$. This parameter is first determined with a value large enough in order to ensure at the end of the algorithm that the partition constrainst is satisfied. The results are not sensitive to variations of $\lambda$, provided it is large enough. Second, the regularization parameter $\gamma$. Variations of $\gamma$ give more or less regular solutions. This parameter is tuned by trial and error.

**Initialization:** To get an automatic initialization, and to make it independent of the user, we have used "seeds": we split the initial image into small sub-images (in practice 5*5 images). In each sub-image, for each class $k$, we compute the data term by assuming that all the pixels of the sub-image belong to the same class $k$. We set all the pixels in the sub-image to the class $k$ for which the whole sub-image's energy is the smallest. We have used this initialization in the examples presented here-after.

### 5.2. Examples:

In case (a) of figure 1, one sees clearly that our model can handle with triple junctions (whereas for instance [4] shows no result of this type). The other cases show that our model can handle with any kind of geometrical shape.

## 6. CONCLUSION

We have presented a variational model based on a level set formulation to classify textured images. The proposed algorithm produces segmention of an entire image according to a piori known texture parameters, by using level sets allowing multiple junctions. The level sets evolve according to wavelet coefficients and interacts between each other.
In a future work, we plan to do a complete theoretical study of our model, and to combine textured and non textured regions in a single classification process.

## 7. REFERENCES


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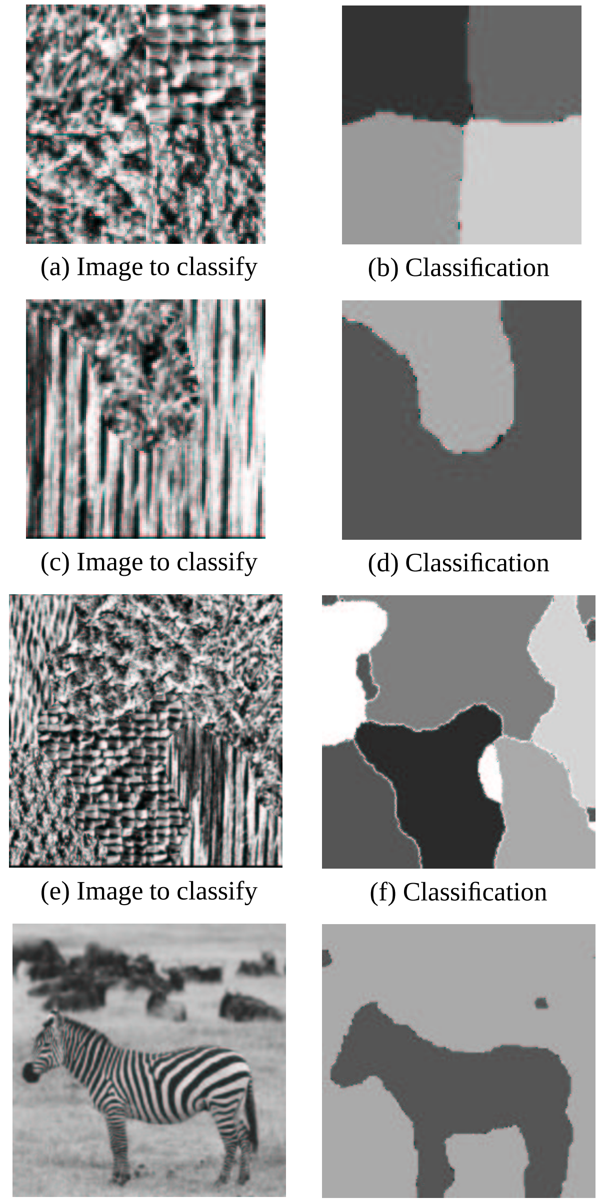

(a) Image to classify (b) Classification

(c) Image to classify (d) Classification

(e) Image to classify (f) Classification

(g) Image to classify (*) (h) Classification

**Fig. 1**. Examples of classification of textured images