CORBIS: Code Raréfié Bidimensionnel Implicite Stationnaire

- main ingredients: [LM (M3AS 00, JCP 00)]
  - plane flow: 2D BGK Model
  - conservative and entropic velocity discretization
  - space discretization: finite volume, curvilinear grids
  - time discretization: backward Euler (transient solutions), linearized implicit scheme (steady flows)

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- new features (2007) : [Aoki-Degond-LM (JCP 07)]
  - reduced distribution technique: $v \in \mathbb{R}^2$ instead of $\mathbb{R}^3$
  - implicit boundary conditions (faster convergence to steady state)
- various boundary conditions can be used on any arbitrary part of the boundaries of domain (diffuse reflection, symmetry axis, periodic condition, out/in-flow condition, temperature gradient on solid wall, evaporation/condensation)
- parallel implementation (Open-MP)
- tested on 16 processors of the Altix 3700 (SGI)

Numerical method: *reduced distribution technique*

- F is independent of $z \Rightarrow$ the transport operator does not contain explicitely the velocity v_z.
- define the reduced distribution function $f(t, x, y, v_x, v_y) = \int_{\mathbb{R}} F\, dv_z$, and integrate BGK w.r.t v_z

$$\partial_t F + v \cdot \nabla_x F = \nu(\mathcal{M}[\rho, u, T] - F)$$

$$\Downarrow \qquad \int_{\mathbb{R}} .\, dv_z$$

$$\partial_t f + v \cdot \nabla_x f = \nu(M[\rho, u, T] - f),$$

where $M[\rho, u, T]$ is the reduced Maxwellian defined by

$$M[\rho, u, T] = \int_{\mathbb{R}} \mathcal{M}[\rho, u, T]\, dv_z = \frac{\rho}{2\pi RT} \exp\left(-\frac{(v_x - u_x)^2 + (v_y - u_y)^2}{2RT}\right),$$

but T cannot be defined through f only:

$$\begin{aligned} E &= \tfrac{1}{2}\rho|u|^2 + \tfrac{3}{2}\rho RT \\ &= \int_{\mathbb{R}^3} \tfrac{1}{2}|v|^2 F(t,x,v)\,dv \\ &= \int_{\mathbb{R}^3} \tfrac{1}{2}|v_x^2 + v_y^2 + v_z^2| F(t,x,v)\,dv \\ &= \int_{\mathbb{R}^2} \tfrac{1}{2}|v_x^2 + v_y^2| f(t,x,v)\,dv_x dv_y + \int_{\mathbb{R}^2} g(t,x,v)\,dv_x dv_y \end{aligned}$$

where $g(t,x,y,v_x,v_y) = \int_{\mathbb{R}} \tfrac{1}{2} v_z^2 F\,dv_z$.

- as for f, an equation for g is derived
- finally, we get the coupled system of kinetic equations:

$$\partial_t f + v \cdot \nabla_x f = \nu(M[\rho, u, T] - f),$$

$$\partial_t g + v \cdot \nabla_x g = \nu(\tfrac{RT}{2} M[\rho, u, T] - g),$$

and the macroscopic quantities are obtained through f and g by

$$\rho = \int_{\mathbb{R}^2} f \, dv^2, \qquad \rho u = \int_{\mathbb{R}^2} v f \, dv^2,$$

$$\frac{1}{2}\rho|u|^2 + \frac{3}{2}\rho RT = \int_{\mathbb{R}^2} (\tfrac{1}{2}|v|^2 f + g) \, dv^2.$$

Numerical method: *velocity discretization*

- for given ρ, u, T, the Maxwellian $M[\rho, u, T]$ satisfies

conservation: $\displaystyle\int_{\mathbb{R}^2} \begin{pmatrix} 1 \\ v \\ \frac{1}{2}|v|^2 \end{pmatrix} M[\rho, u, T]\, dv = \begin{pmatrix} \rho \\ \rho u \\ \frac{1}{2}\rho|u|^2 + \rho RT \end{pmatrix}$

entropy: $\displaystyle\int_{\mathbb{R}^2} M[\rho, u, T] \log M\, dv = \min \left\{ \int_{\mathbb{R}^2} f \log f\, dv \right\}$

- $\mathbb{R}^2$ is truncated to $[v_{\min}, v_{\max}]^2$ and discretized by $(v_k)_{k=1}^N$
- $\displaystyle\int_{\mathbb{R}^2} f\, dv$ is replaced by $\displaystyle\sum_{k=1}^N f_k \Delta v$
- we can define $(M_k)_{k=1}^N$ that satisfies discrete conservation and entropy properties ($\Rightarrow$ existence and convergence results)

equation for f: finite volumes, upwind scheme, curvilinear grid

$$\partial_t f + v \cdot \nabla_x f = \nu(M[\rho, u, T] - f),$$

$$\downarrow$$

$$\partial_t f_{\mathrm{k},i,j} + \frac{1}{\Delta x}(\phi_{i+\frac{1}{2},j}(f_{\mathrm{k}}) - \phi_{i-\frac{1}{2},j}(f_{\mathrm{k}})) + \frac{1}{\Delta y}(\phi_{i,j+\frac{1}{2}}(f_{\mathrm{k}}) - \phi_{i,j-\frac{1}{2}}(f_{\mathrm{k}}))$$
$$= \nu_{i,j}(M_{\mathrm{k}}[\rho_{i,j}, u_{i,j}, T_{i,j}] - f_{\mathrm{k},i,j}),$$

where the numerical fluxes are defined by

$$\phi_{i+\frac{1}{2},j}(f_{\mathrm{k}}) = \frac{1}{2}\Big(v_{x,k}(f_{\mathrm{k},i+1,j} + f_{\mathrm{k},i,j}) - |v_{x,k}|(\Delta f_{\mathrm{k},i+\frac{1}{2},j} - \Phi_{\mathrm{k},i+\frac{1}{2},j})\Big)$$

$$\phi_{i,j+\frac{1}{2}}(f_{\mathrm{k}}) = \frac{1}{2}\Big(v_{y,k}(f_{\mathrm{k},i,j+1} + f_{\mathrm{k},i,j}) - |v_{y,k}|(\Delta f_{\mathrm{k},i,j+\frac{1}{2}} - \Phi_{\mathrm{k},i,j+\frac{1}{2}})\Big)$$

Numerical method: *time discretization*

transient solutions: first order backward euler

$$
\begin{aligned}
\frac{1}{\Delta t}(f_{\mathrm{k},i,j}^{n+1} - f_{\mathrm{k},i,j}^{n}) \quad &+ \quad \frac{1}{\Delta x}(\phi_{i+\frac{1}{2},j}(f_{\mathrm{k}}^{n}) - \phi_{i-\frac{1}{2},j}(f_{\mathrm{k}}^{n})) \\
&+ \quad \frac{1}{\Delta y}(\phi_{i,j+\frac{1}{2}}(f_{\mathrm{k}}^{n}) - \phi_{i,j-\frac{1}{2}}(f_{\mathrm{k}}^{n})) \\
&\qquad = \nu_{i,j}^{n}(M_{\mathrm{k}}[\rho_{i,j}^{n}, u_{i,j}^{n}, T_{i,j}^{n}] - f_{\mathrm{k},i,j}^{n})
\end{aligned}
$$

stability if

$$
\Delta t \leq \frac{1}{\max_{i,j}(\nu_{i,j}^{n})} \quad \text{and} \quad \frac{\Delta t}{\Delta x} \leq \frac{1}{\max_k |v_k|}
$$

restrictive condition for: rapid or dense flows, and steady state steady solutions: forward euler (implicit)

$$\begin{aligned}\frac{1}{\Delta t}(f_{\mathrm{k},i,j}^{n+1} - f_{\mathrm{k},i,j}^{n}) \quad &+ \quad \frac{1}{\Delta x}(\phi_{i+\frac{1}{2},j}(f_{\mathrm{k}}^{n+1}) - \phi_{i-\frac{1}{2},j}(f_{\mathrm{k}}^{n+1})) \\ &+ \quad \frac{1}{\Delta y}(\phi_{i,j+\frac{1}{2}}(f_{\mathrm{k}}^{n+1}) - \phi_{i,j-\frac{1}{2}}(f_{\mathrm{k}}^{n+1})) \\ &\qquad = \nu_{i,j}^{n}(M_{\mathrm{k}}[\boldsymbol{\mu}_{i,j}^{n+1}] - f_{\mathrm{k},i,j}^{n+1})\end{aligned}$$

then linearization:

$$M_{\mathrm{k}}[\boldsymbol{\mu}_{i,j}^{n+1}] \approx M_{\mathrm{k}}[\boldsymbol{\mu}_{i,j}^{n}] + \partial_{\boldsymbol{\mu}} M_{\mathrm{k}}[\boldsymbol{\mu}_{i,j}^{n+1}](\boldsymbol{\mu}_{i,j}^{n+1} - \boldsymbol{\mu}_{i,j}^{n})$$

where $\boldsymbol{\mu} = (\rho, \rho u, \frac{1}{2}\rho|u|^2 + \frac{3}{2}\rho RT)$

Numerical method: *time discretization*

δ-matrix form of the scheme: set $U^n = (\{f^n_{\mathrm{k},i,j}\}_{\mathrm{k},i,j}, \{g^n_{\mathrm{k},i,j}\}_{\mathrm{k},i,j})$
Then the scheme is

$$\left(\frac{I}{\Delta t} + T + B + R^n\right)\delta U^n = RHS^n,$$

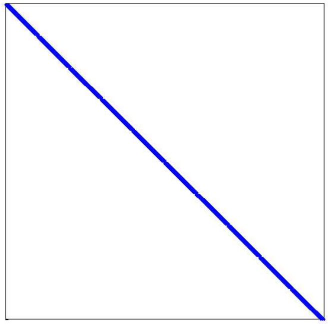
where

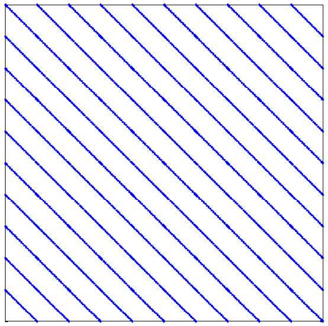
- $\delta U^n = U^{n+1} - U^n$,
- I is the unit matrix,
- T contains the transport coeffi cients, (b. c. in in B)
- B contains the boundary condition coeffi cients,
- R^n is the Jacobian matrix of the collision operator,
- RHS^n is the residual (transport and collision operators applied to U^n).

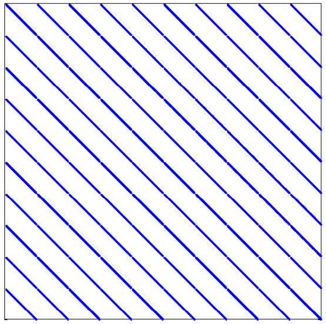
Numerical method: *linear system*

$$\left(\frac{I}{\Delta t} + T + B + R^n\right)\delta U^n = RHS^n,$$

- very large linear system
- sparse matrices

$T =$ 

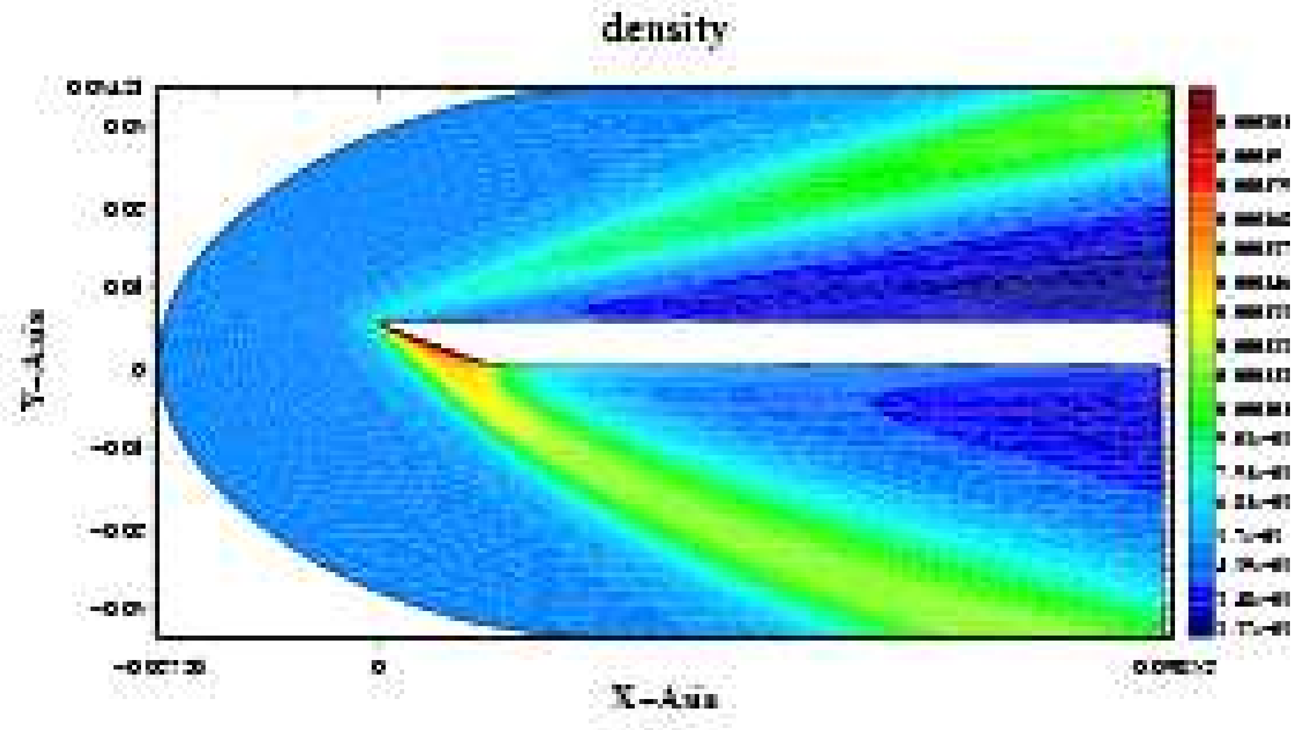
$B =$ 

$R^n =$ 

- an adapted iterative solver is used

Hypersonic external flow:

Mach =18.3, Kn = 0.014

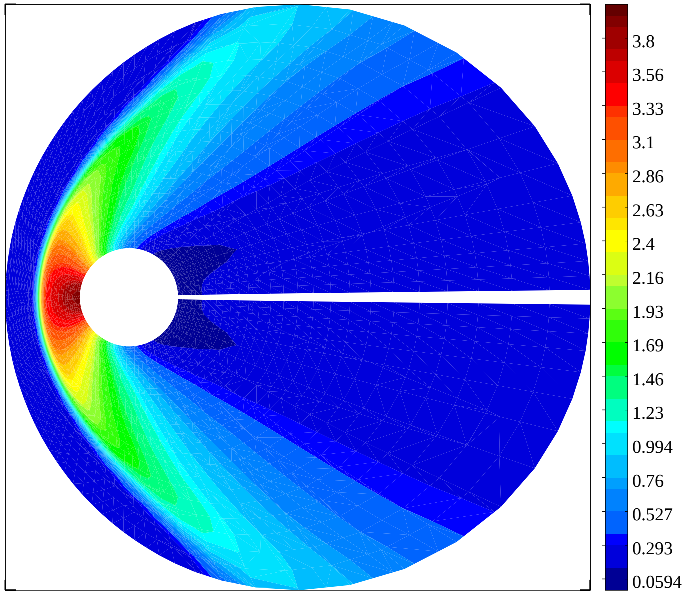


Gallery

Supersonic external flow:

Mach 4, Kn = 0.0358

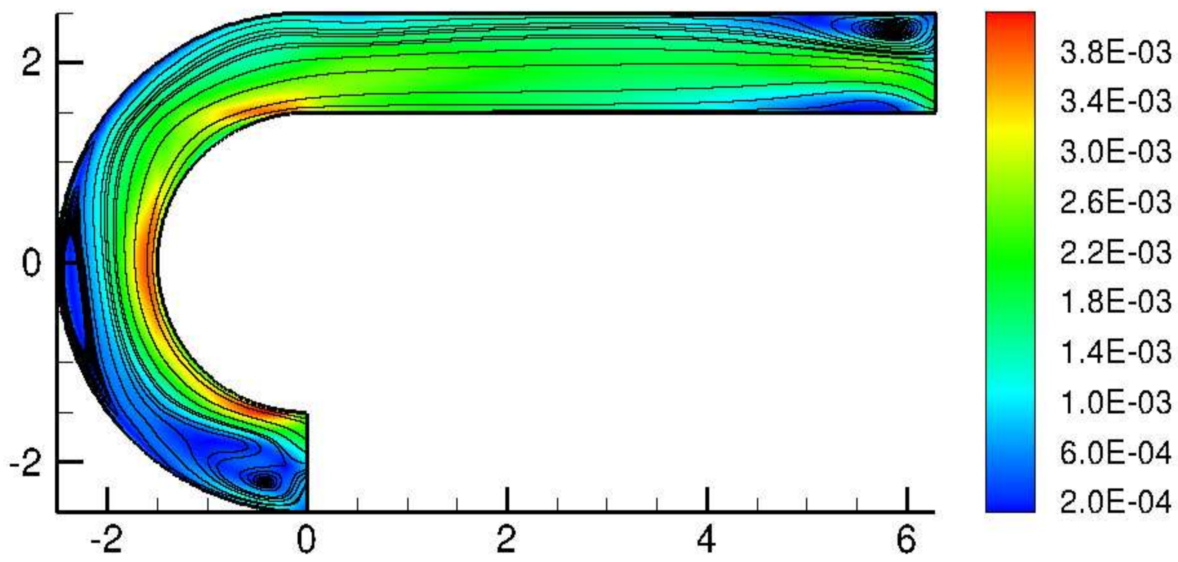
pressure around a cylinder



Internal slow flow:

Mach 0.001, Kn=0.5

Velocity magnitude and streamlines in half of a ring-shaped channel

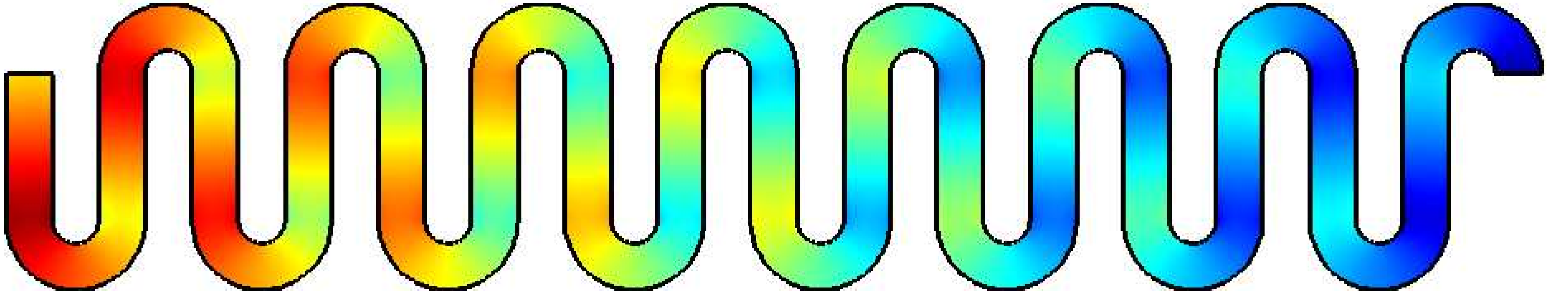


Gallery

Internal slow flow:

Mach 0.001, Kn=0.5

pressure in a micro-pump



Perspectives

- 3D code
- unstructured 2D code
- include polyatomic effects