Errata to the paperback edition 2006

Acknowledgements to: Menny Aka, Soumya Bhattacharya, Martin Brandenburg, Eric Brussel, Edvard Fagerholm, David Harari, Mohammad Hadi Hedayatzadeh, Timo Keller, Shoumin Liu, Yogesh More, Cédric Pépin, Alexander Schmidt. Special thanks to Carlos Ivorra.

Preface, Page x, line 4: replace "Oliver Dodane" by "Olivier Dodane".

Page 9, line 4: Proposition **1**.12.

Page 18, 4 lines above Example 3.4: Delete the assertion "Then any submodule filtration (M_n) of M defines the structure of a topological A-module on M."

Page 41, line 2: replace "open immersion" by "immersion".

Page 63, Proof of Proposition 2.4.9: the ideals in the set S must be **proper** ideals.

Page 66, Exercice 4.6: define an *indecomposable* idempotent element e by the property that for any idemptent element f of A, fe equals to 0 or e. This is equivalent to say that A_e has no other idemptent element than 1. The original definition fails if A has positive characteristic.

Page 92, line 15: $H(S) \in \overline{k}(T_1, \dots, T_d)[S]$ and not $\overline{k}[S]$.

Page 97, line 2: replace $f \times_{\overline{k}}$ by $f_{\overline{k}}$.

Pages 97-98, Exercises 2.14 and 2.15: In 2.14, one can not reduce to the case k algebraically closed as $\overline{k} \otimes_k K$ is in general not a field. Also the connectedness of X_K can not be proved using the function field $K(X)$. See the new text.

Page 98, Exercise 2.17: suppose Y is irreducible and Noetherian.

Page 115, Definition 1.2: the most interesting Dedekind schemes are noetherian instead of locally noetherian. So we will take rather as definition of a Dedekind scheme a normal **noetherian** scheme of dimension 0 or 1. In fact, the noetherian hypothesis is already used in several places (e.g. Proposition 8.3.11, Theorem 8.3.50 etc).

Page 116, Example 1.7: replace "open subset U" by "affine open subset U".

Page 117, line 4: replace "max" by "min".

Page 118, line 5: replace "$\dim A = 0$, see Lemma 2.5.11" by "$\mathfrak{m} = 0$ by Nakayama's lemma, and $\dim A = 0$".

Page 118, lines 12-15 (second part of the proof of Proposition 4.1.12): replace by "Let us show that A is a principal ideal domain. Suppose the contrary. Let I be a maximal element of the set of non-principal ideals of A. Then $x^{-1}I$ is an ideal of A, containing strictly I. So $x^{-1}I$ is principal. But I is then principal, contradiction."

Page 123, line 7: replace "integral over A" by "integral over $A[T]$".

Page 126, §4.2.1: to define the tangent map $T_{f,x}$, we have to suppose either $T_{Y,y}$ has finite dimension over $k(y)$ (e.g. Y is locally Noetherian) or f is locally of finite type. The point is that in general the $k(x)$-dual of $(\mathfrak{m}_y/\mathfrak{m}_y^2)\otimes_{k(y)} k(x)$ is not $T_{Y,y}\otimes_{k(y)} k(x)$.

Pages 127-128, Proposition 2.5: Add the statement $\dim T_{X,x} \le \dim(D_x I)^{\perp}$ for non-rational points. See the new text.

Pages 129, Definition 2.14: replace "system of parameters" by "**regular** system of parameters".

Page 142, Definition 3.35, 5th line: replace "closed points $y \in Y$" by "points $y \in Y$".

Page 144, Exercice 3.2: replace "every closed point $y \in Y$" by "every point $y \in Y$".

Page 149, line -11: replace "finite sub-A-algebra N" by "finitely generated sub-A-module N".

Page 153, bottom line: replace "$\mathfrak{p}$" by "$\mathfrak{m}$".

Page 154, proof of Lemma 4.15: replace $\mathfrak{m}$ by tA and $\mathfrak{m}B$ by tB.

Page 166, line 1: change T_n to T_d.

Page 178, Exercise 5.1.33(b): ρ is the projection $X \times_Y \mathrm{Spec}\mathcal{O}_{\mathrm{Y,y}} \to \mathrm{X}$.

Page 188, line -2: change 1.4(a) to 1.4.

Page 190, Proposition 5.2.34: $f : X \to Y$ must be projective. See the new text.

Page 201, lines 1-2: The hypothesis are X integral and Y normal.

Page 224, Corollary 2.12: Denote by f the structural morphism $X \to S$.

Page 241, in the last displayed formula of the proof of 6.4.12: add the sign $(-1)^{r(n-r)+(n-r)(n-r+1)/2+j_{r+1}+\dots+j_n}$ to Δ_S.

Page 249, Exercise 4.7(b): The existence of some $e \in E$ such that $\mathrm{Tr}_{\mathrm{E/K}}(\mathrm{e}) = 1$ does not imply that E is étale (except when E is a field).

Page 256, line -11: replace "$H^0(X, \mathcal{O}_X \cap \mathcal{K}_X^*)$" by "$H^0(X, (\mathcal{O}_X \cap K_X^*)/\mathcal{O}_X^*)$".

Page 257, line 2: A Cartier divisor is principal if it can be represented by a system $\{(U_i, f_i)_i\}$ such that $f_i|_{U_i\cap U_j} = f_j|_{U_i\cap U_j}$ for all i, j.

Page 282, proof of Lemma 7.3.30: we don't have to make base change to an infinite field K, because Proposition 7.1.32 is now stated over any (noetherian) affine base scheme.

Page 297, last commutative diagram: replace "$G \to G \times_S G \to G \times_S G$" by "$(\mathrm{Id}_G, \mathrm{inv}) : G \to G \times_S G$" to make it shorter.

Page 303, Lemma 7.5.2(a): replace "$\mathrm{Frac}(\mathcal{O}_{\mathrm{X},\xi_\mathrm{i}})$" by "$\mathcal{O}_{X,\xi_i}$" as the later is already a field.

Page 304, top of the page: the proof of the surjectivity of ρ' is insufficient. We want to prove that for any $i \leq n$, $\mathrm{Frac(A/\mathfrak{p}_i)}$ is in the image of ρ'. We can take $i = 1$. Let $a \in A \setminus \mathfrak{p}_1$. Then $\mathfrak{p}_1 + a \cap_{2 \leq i \leq n} \mathfrak{p}_i \not\subseteq \cup_{1 \leq i \leq n} \mathfrak{p}_i$. So there exist $x \in \mathfrak{p}_1$, $b \in \cap_{2 \leq i \leq n} \mathfrak{p}_i$ such that $u := x + ab$ is a regular element of A and we have $\rho'(b/u) = (1/a, 0, \cdots, 0) \in \oplus_{1 \leq i \leq n} \mathrm{Frac(A/\mathfrak{p}_i)}$.

Page 304, Definition 5.3: As example of birational morphisms, the normalization maps are birational, if they are finite (because we only talk about morphisms of finite type).

Page 331, Exercice 8.1.5: Blowing-up is not necessary (it is used to proved that when X is irreducible, then there exists an irreducible curve in X passing through x_1, x_2. See Mumford [71], page 56). Delete (a) and (b). In (c), replace Z by the support of a suitable ample divisor in X.

Page 333, line 4: replace "$\mathbb{A}^n_X$" by "$\mathbb{A}^n_{\mathbb{Z}} \times_{\mathrm{Spec}\mathbb{Z}} X$".

Page 335, Corollary 2.8: We must assume that the generic fiber X_ξ is equidimensional (otherwise the conclusion is false). See the new text.

Page 335, Example 2.10: The statement is incorrect. See the new text.

Page 337, step (δ) of the proof of Proposition 2.13: if $\mathrm{depth} M = 0$, then the inequality to proof is obvious. On the other hand $\mathrm{depth} M = 0$ is equivalent to $\mathfrak{m} \in \mathrm{Ass(M)}$ instead of $\mathrm{Ass(M)} \subseteq \{\mathfrak{m}\}$. In the last displayed formula, replace A/q by $A/\mathfrak{q}$.

Page 339, Corollary 2.25: Add X is connected. Remove "of finite type" for f (included in the definition of smooth morphisms).

Page 350, proof of Corollary 8.3.6 (b) and (c): replace "$\mathcal{O}_X(X_\eta)$" by "$\mathcal{O}_{X_\eta}(X_\eta)$".

Page 354, Definition 3.17, line 12: the morphism f must map the closed point of $\mathrm{Spec}\mathcal{O}_\nu$ to that of $\mathrm{Spec}\mathcal{O}_{\mathrm{X,x}}$. Note that every S-valuation of $K(X)$ has a **unique** center in X because X is separated over S.

Page 361, second line in Theorem 8.3.42: replace "reduce scheme" by "reduced scheme".

Page 364, proof of (b): the first sentence is false in general. See the new text.

Page 364, proof of (d): replace the first displayed formula (which is correct) by the following (maybe more natural) one :

$$\mathcal{O}_X(U) \otimes_R \widehat{R} \subseteq \mathcal{O}_X(U) \otimes_R \widehat{K} = \mathcal{O}_{X_K}(U_K) \otimes_K \widehat{K} = \mathcal{O}_{X_{\widehat{K}}}(U_{\widehat{K}}).$$

Page 365, proof of Theorem 8.3.50, (ii) $\Longrightarrow$ (iii) : the proof of the finiteness of the normalization morphism $X_1 \to X$ is not correct (the finiteness above $\mathcal{O}_{S,s}$ for all s is not enough). See the new text which also contains more details on the proof of (i) $\Longrightarrow$ (ii).

Page 414, Lemma 9.3.6 (d): the ideal $\mathfrak{m}$ is defined in the proof of (b).

Page 414, proof of (a): replace "$\mathcal{J}^k \otimes_{\mathcal{O}_X} \mathcal{O}_E$" by "$\mathcal{J}^k \otimes_{\mathcal{O}_U} \mathcal{O}_E$", and "$H^1(X, .)$"

by “$H^1(U, .)$” (four times).

Page 414, proof of (b): the equality $\sqrt{\mathfrak{m}\mathcal{O}_U} = \mathcal{J}$ comes from that fact that $\mathcal{J}$ defines the structure of a reduced subscheme on E (this is a an exercise that one can solve using the fact that X is locally factorial).

Page 419-420, proof of Proposition 3.16: the new proof is completely independent on the Exercise 4.3.22.

Page 423, line 6 of the proof of 3.27: replace $V \in d_s X_s + V'$ by $V = d_s X_s + V'$.

Page 439, Example 9.4.19: the connectedness hypothesis is not necessary.

Page 444, Lemma 9.4.29(b) and bottom line: read $\pi_* \mathcal{O}_W(n\mathbf{O})$ instead of $\pi_* \mathcal{O}_W(n)$.

Page 448, second line of the proof of (d): replace $\omega_{X'/S}$ by $\omega_{X/S}$. Some lines below: SuppD is both equal to the **exceptional locus of** $X' \to X$ (Exercise 2.4).

Page 449, top line: the factorization theorem is not needed.

Page 449, proof of Corollary 9.4.38: the reference to Example 4.19 is unnecessary.

Page 474, line -2: replace “$C \cdot D$” by “$(C \cdot D)^2$”.

Page 485, the line above the displayed formula (2.14): delette $= [k(\Gamma_i) \cap \overline{k} : k]$ (which is false in general if Γ_i is singular). In the line after, remove reference to Exercise 9.2.8. Actually, Γ_i is a curve defined over $H^0(\Gamma_i, \mathcal{O}_{\Gamma_i})$, so r_i divides $\Gamma_i \cdot \Gamma_j$.

Page 486, line -2: replace “$d_2 = \Gamma_1 \cdot \Gamma_2 = 1$” by “$d_2 = i_p(\Gamma_1, \Gamma_2) = 1$”.

Page 488, line 3 below Figures 36 and 37: replace “$\Gamma_i \cap \Gamma_2 \neq \emptyset$” by “$\Gamma_i \cap \Gamma_3 \neq \emptyset$”.

Page 489, line -5: Proposition **1**.8.

Page 490, lines 1-2: remove “of finite type” as it is included in the smoothness definition.

Page 493, proof of Part (c). We use Proposition 9.3.28 to reduce to the case when S is the spectrum of a complete discrete valuation ring with separably closed residue field. This poses two problems. First Proposition 9.3.28 is written only for finite étable base change; secondly, we didn’t mention the existence of the strict henselization. So we prove (c) by using only finite étale base change and completions. See the new text.

Page 494, proof of Theorem 2.14: Remove “of finite type” (redundant with smoothness). As the connected components of X are integral, we can reduce to the case when X itself is smooth and integral.

Page 495, proof of Lemma 2.17: To prove the finiteness of A^{et} over k, we can extend k to $\overline{k}$ and see that $\dim_k A^{et}$ is bounded by the number of connected components of SpecB.

Page 497, second table: it includes the types I_{2n}^*, I_{2n+1}^* for $n = 0$. In the line above, add Remark 4.12 before Exercise 4.7(b).

Page 504, Exercise 2.5(b): the intersection of the graphs has codimension 2 and is isomorphic to an union of irreducible components of $\mathcal{N}_s$.