CEREDNIK-DRINFELD'S MODELS OF SHIMURA CURVES AND REDUCTION OF CM-POINTS

Let B be an indefinite quaternion algebra over $\mathbb{Q}$ of reduced discriminant D and let $\mathcal{O}$ be an Eichler order of B of level $N \geq 1$, $(D, N) = 1$. Let $X_0(D, N)/\mathbb{Q}$ be the Shimura curve which arises as the coarse moduli space of abelian surfaces with multiplication by $\mathcal{O}$.

Given an order R in an imaginary quadratic field K, there exists a set of points $\mathrm{CM}(R)$ in $X_0(D, N)$, rational over the ring class field H_R of R. Each point $P \in \mathrm{CM}(R)$ is associated with a conjugation class of optimal embeddings $R \hookrightarrow \mathcal{O}$. The Galois action of $\mathrm{Gal}(H_R/K)$ on $\mathrm{CM}(R)$ is described via the Shimura reciprocity law by an explicit action of $\mathrm{Pic}(R)$ on such embeddings.

The aim of this talk is to study the reduction of CM-points at the primes of bad reduction $p \mid D$ of $X_0(D, N)$. Using the moduli interpretation of $X_0(D, N)$ we give necessary and sufficient conditions to determine whether the points in $\mathrm{CM}(R)$ reduce to singular points on the closed fibre $\widetilde{X}_0(D, N)$ of Cerednik-Drinfeld's model of $X_0(D, N)$ over $\mathbb{Z}_p$. More precisely, with the above notation we show:

Theorem. A CM-point $P \in CM(R)$ of $X_0(D, N)$ reduces to a singular point of $\widetilde{X}_0(D, N)$ if and only if p ramifies in K.

The main ingredient of the proof is Ribet's description of Cerednik-Drinfeld's model of $X_0(D, N)$ over $\mathbb{Z}_p$ and its closed fibre, which exploits the classification of abelian surfaces with quaternionic multiplication over $\overline{\mathbb{F}}_p$, in terms of their Dieudonné modules.

Moreover, in case that p ramifies in K, we describe how the action of $\mathrm{Pic}(R)$ on $\mathrm{CM}(R)$ translates into an explicit action on the finite set of singular points of $\widetilde{X}_0(D, N)$. Combined with a recent result of P. Michel, we also show how can one derive an equidistribution result of the Galois orbits of CM-points among the singular points of $\widetilde{X}_0(D, N)$.

As an application, we show how the above circle of ideas is useful in the computation of explicit equations of Shimura curves.