A Random-Projection Based Procedure to Test if a Strictly Stationary Process is Gaussian

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Tests of Gaussianity

Let $X_1, ..., X_n$ be i.i.d. random variables,
then,
we already know Gaussianity tests for this setting!

But, if the r.v.'s are dependent?

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What kind of dependence?

We'll deal with Strictly Stationary Processes

- $\{X_t\}_{t\in\mathbb{Z}}$, ..., $X_1, X_2, \ldots$
- $\{X_t\}_{t\in\mathbb{N}}$, $X_1, X_2, \ldots$

$\{X_t\}_{t\in\mathbb{Z}}$ is a strictly stationary process iff

$$(X_{t_1}, X_{t_2}, \ldots, X_{t_j}) \text{ and } (X_{t_1+k}, X_{t_2+k}, \ldots, X_{t_j+k}) \text{ are i.d for all } k \in \mathbb{Z}$$

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But, there are not already a bunch of tests for this?

Yes,

- Epps' test (1987)
  Check if $\phi_{X_t}(\lambda_i) = \phi_{N(\mu,\sigma^2)}(\lambda_i)$ for $i = 1, ..., N$
- Lobato and Velasco's test (2004)
  Check the skewness and kurtosis of X_t

However, they only test if the marginals of the process are Gaussian.

That is; they test if X_t is Gaussian,
not if $(X_1, ..., X_t)$ is Gaussian

So, is this O.K.?

No, because such tests do not reject non-Gaussian processes with Gaussian marginals

We need a new test that reject such kind of non-Gaussian processes!

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Theorem (Cuesta, Barrio, Fraiman and Matrán, 2007)

Let η be a dissipative distribution on a separable Hilbert space, $\mathbf{H}$. If $(..., X_t)$ is an $\mathbf{H}$-valued random element and

$$\eta\{h \in \mathbf{H} : \text{ the distribution of } \langle (..., X_t), h\rangle \text{ is Gaussian}\} > 0,$$

then X is Gaussian.

It follows,

$\eta\{h \in \mathbf{H} :$ **the distribution of** $\langle (..., X_t), h\rangle$ **is Gaussian**$\} \in \{0, 1\}$,

So, selecting h using a dissipative distribution we have,

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In Practice

- $\boldsymbol{H} = \left\{ (x_n)_{n\in\mathbb{N}^*} : \sum_{n\in\mathbb{N}^*} x_n^2 a_n < \infty \right\}$,
  - $a_n = \min(1, n^{-2})$ and $\mathbb{N}^* = \mathbb{N} \cup \{0\}$
  - $\langle \mathbf{x}, \mathbf{y} \rangle = \sum_{n\in\mathbb{N}^*} x_n y_n a_n$, where $\mathbf{x} = (x_n)_{n\in\mathbb{N}^*}, \mathbf{y} = (y_n)_{n\in\mathbb{N}^*}$

- Dissipative distribution
  - l_0 chosen with $\beta(\alpha_1, \alpha_2)$
  - l_n chosen with $\beta(\alpha_1, \alpha_2)[0, 1 - \sum_{i=0}^{n-1} \eta_i]$, for $n \geq 1$

  $h_n = (l_n / a_n)$ for $n \geq 0$

  $(h_0, \ldots) \in \boldsymbol{H}$ has dissipative distribution

- Fix $h = (h_0, \ldots)$
- Define $Y_t := \sum_{i=0}^{\infty} h_i X_{t-i} a_i$

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$\{Y_t\}_{t\in\mathbb{Z}}$ inherit $\{X_t\}_{t\in\mathbb{Z}}$ properties
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Test Gaussianity of $\{Y_t\}_{t\in\mathbb{Z}}$ with a procedure that check if the marginals of the process are Gaussian

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Simulations

Given an AR(1) process

$$X_t = q * X_{t-1} + \epsilon_t$$

Compare results of

- E, Epps' test (1987)
- G, Lobato and Velasco's test (2004)

	n=100							
q	Test	N(0,1)	log N	t_{10}	χ^2_1	χ^2_{10}	$U(0,1)$	$\beta(2,1)$
	E	.0724	.6780	.0556	.8514	.2058	.5408	.4914
-.5	G	.0364	.9976	.1752	.9992	.4300	.0006	.1174
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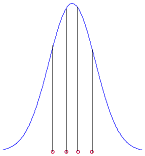
	GE	.1004	.9256	.0936	.7284	.1780	.1174	.1372
	RP	.1000	.8742	.0930	.6316	.9136	.5958	.9894

A Non-Gaussian Process with Gaussian Marginals

A family of this kind of processes, $\mathfrak{F}(p)$, is given in Cuesta and Matrán (1991), with p a prime number.
Let $p = 5$,

$$\dots, \underbrace{X_{mp}, X_{mp+1}, X_{mp+2}, X_{mp+3} X_{mp+4}}, \underbrace{X_{(m+1)p}, \dots, X_{(m+1)p+4}}, \cdots$$

- a strictly stationary process
- of pairwise independent variables
- with X_t Gaussian for all $t \in \mathbb{Z}$
- without mutually independent variables



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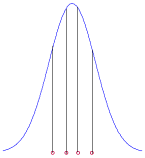

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Summarizing

1. Tests of Gaussianity
2. Gaussianity Tests for Strictly Stationary Processes
3. The Random Projection Test (RP test)
4. Simulations
5. Conclusions

Conclusions

Given a strictly stationary process $\{X_t\}_{t\in\mathbb{Z}}$,

the RP test check if $\{X_t\}_{t\in\mathbb{Z}}$ is Gaussian

Procedure:

- Take $h \in \boldsymbol{H}$ following η
- $Y_t :=< (..., X_t), h >$
- Check if the marginals of the strictly stationary process $\{Y_t\}_{t\in\mathbb{Z}}$ are Gaussian

Advantage:

- Reject non-Gaussian processes with Gaussian marginals

Thank you very much!