CORRECTIONS FOR THE BOOK "ANALYTIC NUMBER THEORY" (COLL. PUBL. 53)

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Thanks to colleagues who have sent corrections, in particular F. Chamizo, É. Fouvry, É. Royer, J-P. Serre and I. Shparlinski.

Introduction, Page 3, Line 6: Typo: "primary" should be "primarily".

Section 1, Page 19, Line -6: Replace roman M by $\mathcal{M}$ in three places.

Section 1, Page 29, Line 6: Write $\mathcal{M}_{f^2}(x)$ instead of $\mathcal{M}_f^2(x)$.

Section 3, Page 48, Line -7: The formula in Lemma 3.2 should have $\tau(\chi^\star)$ on the right-hande side instead of $\tau(\chi)$ as stated. The correct form is used in Chapter 12, page 324.

Section 3, Page 54, Line -11: The equal sign is misplaced in the statement of the quartic reciprocity law (3.56), which should read

$$\Big(\frac{\pi_1}{\pi_2}\Big) = (-1)^{(p_1-1)/4(p_2-1)/4}\Big(\frac{\pi_2}{\pi_1}\Big).$$

Section 3, Page 55, Line 7: The character χ should be $\chi_{\pi_1}^2\cdots\chi_{\pi_r}^2$ (i.e., the square of the one indicated).

Section 3, Page 62, Line 8: In the case $\mathfrak{m} = (1)$, i.e., if $4 \mid \ell$, it should be remarked that the expression in (3.91) is independent of the choice of a generator α of $\mathfrak{a}$, since in that case there may not exist a primary generator.

Section 4, Page 79, Line 7: The correct quotation in **[GR]** is 8.451.2 instead of 23.451.2.

Section 4, Page 87, Line -13: In formula (4.81), the right-hand side should be

$$\frac{1}{Y}\Big(\frac{\sin \pi xY}{\pi x}\Big)^2$$

instead of

$$\Big(\frac{\sin \pi xY}{\pi xY}\Big)^2.$$

Section 5, Page 99, Line -8: Replace the formula by

$$\sum_n n^{1/2}\lambda_f(n)\exp\Big(-\frac{2\pi n}{X\sqrt{q}}\Big) = X^2\varepsilon(f)\sum_n n^{1/2}\overline{\lambda_f(n)}\exp\Big(-\frac{2\pi nX}{\sqrt{q}}\Big).$$

Section 5, Page 105, Line -13: D. Lieman's name is misspelled Liehman here, and also on page 140, line 17, and page 141, line 13 (it is correctly spelled in the bibliography).

Section 5, Page 119, Line 13: The root number of $L(s,\chi)$, as proved in Theorem 4.15, is $i^{-\delta}\tau(\chi)/\sqrt{q}$.

Section 5, Page 130, Line 15: Here and on line 18 and 24, $(\alpha/|\alpha|)^{ik}$ should be $(\alpha/|\alpha|)^k$.

Section 5, Page 130, Line 24: The factor 2 should be 4 (we are counting integers instead of ideals).

Section 5, Page 130, Th. 5.36: The main term should be $2\frac{\beta-\alpha}{\pi}\operatorname{li}(x)$.

Section 5, Page 130, Line -7: In the Hint, x designates the variable for the characteristic function of $[\alpha,\beta]$, not the limit x in Theorem 5.36. Also, the approximation to the characteristic function on $[0,2\pi]$ should be in fact periodic of period $\pi/2$ instead of symmetric with respect to $x \mapsto x \pm \pi$.

Section 5, Page 135, Line 10: There is no exceptional zero for classical modular forms, as shown by J. Hoffstein and D. Ramakrishnan (*Siegel zeros and cusp forms*, Int. Math. Res. Not. 1995, No.6, 279-308 (1995).)

Section 5, Page 137, Th. 5.41: Molteni's result requires an assumption of the type $\lambda_f(p) \ll p^{1/4-\delta}$ for some $\delta > 0$. The implied constant in the convexity bound then further depends on δ.

Section 5, Page 144, Line 16: The error term in the sample asymptotic for $\psi(x, C)$ on GRH is worse than it should be, namely the proof gives

$$\psi(x,C) = \frac{|C|x}{|\mathrm{Gal}(L/K)|} + O\Big(\sqrt{x}(\log x)(\sqrt{|C|}(\log x^{[K:\mathbf{Q}]}) + \frac{|C|}{|\mathrm{Gal}(L/K)|}\log|d_L|)\Big).$$

simply by using the obvious estimate

$$|c_\rho| \leqslant \frac{|C|}{|\mathrm{Gal}(L/K)|}\deg\rho,$$

instead of $|c_\rho| \leqslant \deg\rho$.

Section 5, Page 148, Cor. 5.49, 5.50: In both corollaries, the L-functions involved are assumed to satisfy GRH.

Section 5, Page 149, Line 3,-14,-12,-11,-10: The lower bound in Theorem 5.51 should be $q \geqslant e^{0.8g}$ so $q > 2$. In the proof the Ramanujan-Petersson bound gives

$$\Big|\frac{L'}{L}(A,\sigma)\Big| \leqslant 2g\Big|\frac{\zeta'}{\zeta}(\sigma)\Big|$$

(since the L-function is of degree $2g$), then in line -12, replace $-\zeta'(\sigma)/\zeta(\sigma)$ by $+2\zeta'(\sigma)/\zeta(\sigma)$. Taking $\sigma = 3$ gives that the right-hand side is > 0.4, hence the result.

Section 5, Page 149, Line -9: Since $e^{1.6} < 11$, one can't argue directly from Theorem 5.51 as corrected. However, Mestre [**Mes**] has shown using the explicit formula that $q > 10^g$, which gives the stated result.

Section 7, Page 181, Line -8: The modulus of $\sum X(p,\nu)$ should be squared.

Section 7, Page 188, Line 2: The argument $\sqrt{mn}$ of the Bessel function on the right-hand side of the inequality should be N. On line 5, the value of x should be the upper bound $x = 2\pi Nc^{-1}$.

Section 8, Page 208, Line 4: Replace "g is given by" with "G is given by".

Section 9, Page 231, Line 17: An exponent 2 is missing in $|c_\ell|$ which should be $|c_\ell|^2$.

Section 9, Page 231, Line -6: An exponent 2 is missing in $|a_{n_1}^{(1)}\cdots a_{n_k}^{(k)}|$ which should be $|a_{n_1}^{(1)}\cdots a_{n_k}^{(k)}|^2$.

Section 9, Page 233, Line 6: The closing parenthesis is missing and dt should not be subscripted.

Section 11, Page 280, Line -2,-8: One should replace m and n by a and b respectively.

Section 11, Page 285, Line 6: Replace the formula by

$$\sum_{k\leqslant j<J}(r_j + s_j g)X^{(j-k)q} = 0$$

(i.e. change the k subscript to j in the sum).

Section 11, Page 287, Line 2: Replace the formula by

$$N_1 = q - N_0 - N_2 > \frac{q}{2} - 4m\sqrt{q}$$

(i.e. replace the second N_1 by N_2).

Section 11, Page 305, Definition, l. -16: Replace "are algebraic integers" by "are algebraic numbers".

Section 11, Page 308, Line -16: Remove "only" in "only all the...".

Section 11, Page 309, Line -14 to -9: Replace $K_r(\cdot, p)$ by $K_r(\cdot, q)$ (in six places altogether).

Section 11, Page 309, Line -11: Replace $\mathbb{F}_p$ by $\mathbb{F}_p^\star$.

Section 11, Page 309, Line -11: Replace $K_r(a_0v^2, p)$ by $K_r(a_0v^r, q)$.

Section 12, Page 327, Line -1: The exponent of V_1 should be $1 - \frac{1}{r}$ instead of $1 - \frac{1}{4}$.

Section 15, Page 399, Line -6: In the integral, $k(i,z)$ should be replaced by $k(u(i,z))$.

Section 16, Page 413, Line -4: The correct quotation in [**GR**] is 8.411.4 instead of 23.411.4.

Section 17, Page 421, Line 6: The bound on the right-hand side for the Barban-Davenport-Halberstam theorem should be $x^2(\log x)^{-A}$ instead of $x(\log x)^{-A}$.

Section 21, Page 487, Line 8: The open sets B for which equidistribution is tested should be such that the boundary of B in X has measure 0.

Section 21, Page 490, Line 9: The Weyl criterion reads $S = o(w(p)^n)$ instead of $S \ll w(p)^n$.

Section 22, Page 512, Line 13: The correct quotation in [**GR**] is 8.451.6 instead of 23.451.6.

Section 26, Page 587, Line 13: The correct quotation in [**GR**] is 6.561.14 instead of 14.561.14.

Bibliography, Page 605, Line 6: The reference for the paper of Kim and Sarnak should be [**KiS**], as quoted in Chapters 5 and 15.