Automorphisms and monodromy

C. LEHR [1]   M. MATIGNON[2]

[1]MPI Bonn   [2]Laboratoire A2X Bordeaux

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Monodromy and automorphism groups

Automorphism groups of curves in char.$p > 0$

p -cyclic covers of the affi ne line

Structure of $G_{\infty,1}(f)$

Bounds for $|G_{\infty,1}(f)|$

Characterization of $G_{\infty,1}(f)$

Actions of p-groups over a curve C with $g(C) \geq 2$

Big actions (I)

Condition (N) and G_2

Riemann surfaces

Ray class fi elds

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Maximal curves

Monodromy polynomial

Marked stable model

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Monodromy and automorphism groups.

- R is a strictly henselian DVR of inequal characteristic $(0, p)$.
  $K := \mathrm{Fr}R$; for example $K/\mathbb{Q}_p^{ur}$ finite.
  π a uniformizing parameter.
  $k := R_K/\pi R_K$.
  C/K smooth projective curve, $g(C) \geq 1$.
- C has potentially good reduction over K if there is L/K (finite) such that $C \times_K L$ has a smooth model over R_L. Then:
- There is a minimal extension L/K with this property ; it is Galois and called the **monodromy** extension.
- $\mathrm{Gal}(L/K)$ is the **monodromy group**.
- Its p-Sylow subgroup is the **wild monodromy group** .

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- The base change $C \times_K K^{alg}$ induces an homomorphism $\varphi : \mathrm{Gal}(K^{alg}/K) \to \mathrm{Aut}_k C_s$, where C_s is the special fiber of the smooth model over R_L and $L = (K^{alg})^{\ker \varphi}$.
- Let ℓ be a prime number, then, $n_\ell := v_\ell(|\mathrm{Gal}(L/K)|) \leq v_\ell(|\mathrm{Aut}_k C_s|)$.
- If $\ell \notin \{2, p\}$, then ℓ^{n_ℓ} is bounded by the maximal order of an ℓ-cyclic subgroup of $\mathrm{GL}_{2g}(\mathbb{Z}/\ell\mathbb{Z})$ i.e. $\ell^{n_\ell} \leq O(g)$.
- If $p > 2$, then $n_p \leq \inf_{\ell \neq 2,p} v_p(|\mathrm{GL}_{2g}(\mathbb{Z}/\ell\mathbb{Z})|) = a + [a/p] + \ldots$, where $a = [\frac{2g}{p-1}]$.
  This gives an exponential type bound in g for $|\mathrm{Aut}_k C_s|$. This justifies our interest in looking at Stichtenoth ([St 73]) and Singh ([Si 73]).

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Theorem

([Ra 90]). Let $Y_K \to X_K$ be a Galois cover with group G. Let us assume that:

- *G is nilpotent.*
- *X_K has a smooth model X.*
- *The Zariski closure B of the branch locus B_K in X is étale over R_K.*

Then,

- *the special fiber of the stable model Y_K is tree-like, i.e. the Jacobian of Y_K has potentially good reduction.*

- Raynaud's proof is qualitative and it seems difficult to give a constructive one in the simplest cases.
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p-cyclic covers of the affine line

k is an algebraically closed of char. $p > 0$.

- $f(X) \in Xk[X]$ monic,deg $f = m > 1$ prime to p, monic.
- $C_f : W^p - W = f(X)$. Let ∞ be the point of C_f above $X = \infty$ and z a local parameter. Then, $g := g(C_f) = \frac{p-1}{2}(m-1) > 0$.
- $G_\infty(f) := \{\sigma \in \mathrm{Aut}_k C_f \mid \sigma(\infty) = \infty\}$.
- $G_{\infty,1}(f) := \{\sigma \in \mathrm{Aut}_k C_f \mid v_\infty(\sigma(z) - z) \geq 2\}$, the p-Sylow.
- ([St 73]) Let $g(C_f) \geq 2$, then $G_{\infty,1}(f)$ is a p-Sylow of $\mathrm{Aut}_k C_f$.
- It is normal except for $f(X) = X^m$ where $m | 1 + p$.

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Structure of $G_{\infty,1}(f)$

- Let $\rho(X) = X$, $\rho(W) = W + 1$, then $<\rho>= G_{\infty,2} \subset Z(G_{\infty,1})$
- $0 \to <\rho> \to G_{\infty,1} \to V \to 0$,
  $V = \{\tau_y | \ \tau_y(X) = X + y,\ y \in k\}$.
  $f(X+y) = f(X) + f(y) + (F - \mathrm{Id})(P(X,y))$,
  $P(X,y) \in Xk[X]$.
  $V \simeq (\mathbb{Z}/p\mathbb{Z})^v$ as a subgroup of k.
- Let $\tau_y(W) := W + a_y + P(X,y)$, $a_y \in \mathbb{F}_p$, then $[\tau_y, \tau_z] = \rho^{\epsilon(y,z)}$, where $\epsilon : V \times V \to \mathbb{F}_p$ is an alternating form.
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Bounds for $|G_{\infty,1}(f)|$

Lemma

If $f(X) = \sum_{1 \le i \le m} t_i X^i \in k[X]$ *is monic, then:*

- $\Delta(f)(X,Y) := f(X+Y) - f(X) - f(Y) = R(X,Y) + (F - \mathrm{Id})(P_f(X,Y))$, *where* $R \in \bigoplus_{\lfloor \frac{m}{p} \rfloor \le ip^{n(i)} < m,\ (i,p)=1} k[Y]X^{ip^{n(i)}}$ *and* $P_f \in Xk[X,Y]$.
- $P_f = (\mathrm{Id} + F + ... + F^{n-1})(\Delta(f)) \mod X^{[\frac{m-1}{p}]+1}$.
- *Let us denote by* $\mathrm{Ad}_f(Y)$ *the content of* $R(X,Y) \in k[Y][X]$.
- $\mathrm{Ad}_f(Y)$ *is an additive and separable polynomial.*
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Let $m-1=\ell p^s$ with $(\ell,p)=1$.

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- We consider the extensions of type $0 \to N \simeq Z/p\mathbb{Z} \to G \to (\mathbb{Z}/p\mathbb{Z})^n \to 0$ (note that $G_{\infty,1}(f)$ is an extension of this type). Then $G' \subset N \subset Z(G)$.
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  ([Le-Ma 1]). Let $f(X) \in Xk[X]$ with $(\deg f, p) = 1$. If $\frac{|G_{\infty,1}|}{g} > \frac{p}{p-1}$ ($\frac{2}{3}$ for $p = 2$), then $f(X) = cX + X\Sigma(F)(X) \in k[X]$.
- Sketch proof: One shows that monomials in f with a degree $\notin 1 + p^{\mathbb{N}}$ will limit the degree of Ad_f.
- Let (C, G) with $G \subset \mathrm{Aut}_k C$. We say that (C, G) is a **big action** if G is a p-group and
  (N) $g_C > 0$ and $\frac{|G|}{g_C} > \frac{2p}{p-1}$.
  It follows from ([Na 87]) that there is $\infty \in C$, with
  - $C \to C/G \simeq \mathbb{P}^1_k - \infty$ is étale and $G = G_{\infty,1}$.
  - $G_{\infty,2} \neq G_{\infty,1}$ and $C/G_{\infty,2} \simeq \mathbb{P}^1_k$
  - Then, $G_{\infty,1}/G_{\infty,2}$ acts as a group of translations of the affi ne line $C/G_{\infty,2} - \{\infty\}$.
- **Transfert of condition (N) to quotients.** Let (C, G) a big action, if $H \lhd G$ and if $g(C/H) > 0$, then $(C/H, G/H)$ is a big action.

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Condition (N) and G_2

In this section (C, G) **is a big action.** Let G_i be the lower ramification groups.

- Let $H \lhd G$ and H with index p in G_2 (H exists!), then $(C/H, G/H)$ is a big action.
- $(G/H)_2 = G_2/H \simeq \mathbb{Z}/p\mathbb{Z}$.
- There is $S(F) \in k\{F\}$, $f_1 = cX + X\Sigma(F)(X) \in k[X]$ with $C/H \simeq C_{f_1}$.
- If $G_2 \simeq (\mathbb{Z}/p\mathbb{Z})^t$, then $k(C) = k(X, W_1, ..., W_t)$ and $\wp(W_1, ..., W_t) = (f_1(X), f_2(X), ..., f_t(X)) \in (k[X])^t$
- $f_1(X), .., f_t(X)$ are $\mathbb{F}_p$-free $\mod \wp(k[X])$.
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- $\mathrm{Im}\rho$ is a unipotent subgroup of $\mathrm{Gl}_t(\mathbb{F}_p)$ which is the identity iff $G_2 \subset Z(G)$. In this case $f_i(X) = c_i X + X\Sigma_i(F)(X)$ where $\Sigma_i(F) \in k\{F\}$ and $v \in V$ is a commun zero to the palyndromic polynomials $\mathrm{Ad}_{f_i} \in k\{F, F^{-1}\}$.
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- $\mathrm{Im}\rho$ is a unipotent subgroup of $\mathrm{Gl}_t(\mathbb{F}_p)$ which is the identity iff $G_2 \subset Z(G)$. In this case $f_i(X) = c_i X + X\Sigma_i(F)(X)$ where $\Sigma_i(F) \in k\{F\}$ and $v \in V$ is a commun zero to the palyndromic polynomials $\mathrm{Ad}_{f_i} \in k\{F, F^{-1}\}$.
- Let $f_1 := X(\alpha F)(X) = \alpha X^{1+p}$ with $\alpha^p + \alpha = 0$; then $\mathrm{Ad}_{f_1} = Y^{p^2} - Y$.
- Let $f_2 := X^{1+2p} - X^{2+p}$, then
- $f_2(X+Y) - f_2(X) - f_2(Y) = 2(Y^p - Y)X^{1+p} + (Y - Y^{p^2})X^{2p} + (Y^{2p^2} - Y^2 + 2Y^{1+p} - 2Y^{p+p^2})X^p \mod \wp(k[X,Y])$
- If $y \in Z(\mathrm{Ad}_{f_1}) = \mathbb{F}_{p^2}$ one has $f_2(X+y) = \frac{2(y^p-y)}{\alpha} f_1(X) + f_2(X) + \wp(P_2)$.
- $y \rightarrow \frac{2(y^p-y)}{\alpha}$ is a non zero linear form over $\mathbb{F}_{p^2}$ with value in $\mathbb{F}_p$.
- $|G| = p^2p^2$ and $g = \frac{p-1}{2}(p + p * 2p)$.
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- **Theorem**
  ([Le-Ma 4]) Assume G_2 is non abelian, then $G_2 = G'$.
- Sketch proof: If $G' \neq G_2$, there is $H \lhd G$ with $G' \subset H \subset G_2$ and $[G_2 : H] = p$.
  $(C/H, G/H)$ is abig action ;
- $C/H : W^p - W = f := X\Sigma(F)(X)$, $\deg(f) = 1 + p^s$.
- $(\mathrm{Aut}C/H)_{\infty,1} := E$ is extraspecial with order p^{2s+1}.
- G/H is abelian and normal in E.
- ([Hu 67] Satz 13.7 p. 353) $|G/H| \leq p^{s+1}$ and so $|G/H|/g(C/H) \leq \frac{2p^{s+1}}{(p-1)p^s} = \frac{2p}{p-1}$, a contradiction.
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- In characteristic 0, an analogue of big actions is given by the actions of a finite group G on a compact Riemann surface C with $g_C \geq 2$ such that $|G| = 84(g_C - 1)$ (we say that C is an **Hurwitz curve**) ([Co 90]).
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- Let C be an Hurwitz curve with genus g_C. Let $n > 1$ and C_n the maximal unramified Galois cover whose group is abelian with exponent n. The Galois group of C_n/C is $(\mathbb{Z}/n\mathbb{Z})^{2g_C}$. It follows from the unicity of C_n that the k-automorphisms of C have n^{2g} prolongations to C_n. Therefore $g_{C_n} - 1 = n^{2g}(g_C - 1)$ and $n^{2g}|\mathrm{Aut}_k C| \leq |\mathrm{Aut}_k C_n|$, where $|\mathrm{Aut}_k C_n| \geq 84(g_{C_n} - 1)$; C_n is an Hurwitz curve ([Mc] 61).

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Ray class fields

- If (C, G) is a big action then $C \to C/G$ is an étale cover of the affine line whose group is a p-group ; it follows that the Hasse-Witt invariant of C is zero ; therefore, in order to adapt the previous proof to char. $p > 0$, one needs to accept ramification. This is done with the so called ray class fields of function fields over finite fields.
- Let $K := \mathbb{F}_q(X)$ where $q = p^e$, S the set of finite rational places $(X - v)$, $v \in \mathbb{F}_q$ and $m \in \mathbb{N}$. Let K^{alg} be an algebraic closure. Let $K_S^m \subset K^{alg}$, the biggest abelian extension L of K with conductor $\leq m\infty$ and such that the places in S are completely decomposed.

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- ([Le-Ma 4]) Let $C_m/\mathbb{F}_q$ be the smooth projective curve with function field K_S^m. The translations $X \to X+v,\ v \in \mathbb{F}_q$ stabilize S and ∞; they can be extended to $\mathbb{F}_q$-automorphisms of K_S^m. In this way, we get an action of a p-group $G(m)$ on C_m with $0 \to G_S(m) \to G(m) \to \mathbb{F}_q \to 0$
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  - Let $v \in V := Z(\mathrm{Ad}_{f_0})$ and $P \in k[X]$ with $f_0(X + v) = f_0(X) + \wp(P)$ alors $f_1(X + v) - f_1(X) = \ell(v)f_0(X) + \frac{1}{p}(f_0(X)^p + P(X)^p - P(X)^{p^2} - (f_0(X) + P(X))^p - f_0(X + v)^p + (f_0(X + v) + P(X)^p)^p)$
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- For G_2 abelian with exponent p^e, $e \geq 2$, one can expect for its p-rank a lower bound in $O(log(g_C))$. This is the case in the preceding situation i.e. $(C, G) = (C_{m_2}, G(m_2))$ ([M. Rocher, thesis in preparation]).

Big actions (II)

From now on, k is any algebraically closed field and (C,G) is a big action.

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Maximal curves (I)

Let us assume that (C, G) is a big action.

- Let i_0 with $G_2 = G_3 = ... = G_{i_0} \supsetneq G_{i_0+1}$. Then $g_{(C/G_{i_0+1})} = \frac{1}{2}(|G_2/G_{i_0+1}| - 1)(i_0 - 1)$.
- If $0 < M \leq \frac{|G|}{g_C^2}$, then
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  - $\frac{|G|}{g_C^2} = \frac{4p}{(p-1)^2}$ *and* $G = G_{\infty,1}(f)$ *or*
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We still assume that (C, G) is a big action.

- One can push the “classifi cation ” of big actions up to the condition $\frac{|G|}{g_C^2} \geq \frac{4}{(p^2-1)^2}$. Namely
  - One fi rst show that $|G_2|$ divides p^3.
  - The condition $G_2 = G_1'$ implies that G_2 is abelian.
  - Applying ([Mr 71]) to the case of abelian extensions with group $\mathbb{Z}/p\mathbb{Z} \times \mathbb{Z}/p^2\mathbb{Z}$ one shows that G_2 has exponent p (we have seen above that G_2 is cyclic iff $G_2 = \mathbb{Z}/p\mathbb{Z}$).
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    ([Le-Ma 4]) For all $M > 0$, the set $\frac{|G|}{g_C^2} > M$, for (C, G) a big action with G_2 abelian with exponent p, is finite.

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- If $\mathrm{Im}\rho$ is trivial.
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Monodromy polynomial ([Le-Ma 3])

- Let $C \longrightarrow \mathbb{P}^1_K$ birationally given by the equation: $Z_0^p = f(X_0) = \prod_{1 \leq i \leq m} (X_0 - x_i)^{n_i} \in R[X_0]$, $(n_i, p) = 1$ and $(\deg f, p) = 1$, $v(x_i - x_j) = v(x_i) = 0$ for $i \neq j$.
- $f'(Y)/f(Y) = S_1(Y)/S_0(Y)$, $(S_0(Y), S_1(Y)) = 1$; then $\deg(S_1(Y)) = m - 1$ and $\deg(S_0(Y)) = m$.
- $f(X+Y) = f(Y)((1 + a_1(Y)X + ... + a_r(Y)X^r)^p - \sum_{r+1 \leq i \leq n} A_i(Y)X^i)$, où $r + 1 = [n/p]$, $a_i(Y), A_i(Y) \in K(Y)$.
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$$A_{p^\alpha}(Y) = -\binom{\frac{1}{p}}{p^{\alpha-1}}^p \cdot \frac{S_1(Y)^{p^\alpha} + pT(Y)}{S_0(Y)^{p^\alpha}}.$$
- $\mathcal{L}(Y) := S_1(Y)^{p^\alpha} + pT(Y)$. This is a polynomial of degree $p^\alpha(m-1)$ which is called the *monodromy polynomial* of $f(Y)$.

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Special fiber of the easy model

We mean the R-model $\mathcal{C}_R$ defined by
$Z_0^p = f(X_0) = \prod_{1 \le i \le m} (X_0 - x_i)^{n_i} \in R[X_0]$ (cf. fig 1).

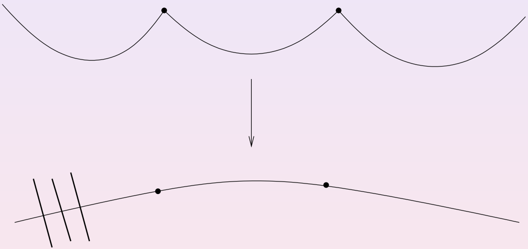

FIG.: $\mathcal{C}_R \otimes_R k \longrightarrow \mathbb{P}^1_k$ with singularities and branch locus

- **Theorem**
  ([Le-Ma 3]) The components with genus > 0 of the marked stable model of C correspond bijectively to the Gauss valuations v_{X_j} with $\rho_j X_j = X_0 - y_j$, where y_j is a zero of the monodromy polynomial $\mathcal{L}(Y)$
- $\rho_j \in R^{\mathrm{alg}}$ *satisfies* $v(\rho_j) = \max\{\frac{1}{i} v\left(\frac{\lambda^p}{A_i(y_j)}\right)$ *for* $r + 1 \leq i \leq n\}$.
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Potentially good reduction with $m = 1 + p^s$

Theorem

([Le-Ma 3])

- $p > 2$, $q = p^n$, $n \geq 1$, $K = \mathbb{Q}_p^{\mathrm{ur}}(p^{p/(q+1)})$ *and* $C \longrightarrow \mathbb{P}^1_K$ *is birationally defined by the equation* $Z_0^p = f(X_0) = 1 + p^{p/(q+1)} X_0^q + X_0^{q+1}$.
- *Then,* C *has potentially good reduction and* $\mathcal{L}(Y)$ *is irreducible over* K.
- *The monodromy* L/K *is the extension of the decomposition field of* $\mathcal{L}(Y)$ *obtained by adjoining the* p*-roots* $f(y)^{1/p}$, *for* y *describing the zeroes of* $\mathcal{L}(Y)$.
- *The monodromy group is the extraspecial group with exponent* p^2 *and order* pq^2 *(which is maximal for this conductor).*

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Curves of genus 2 ([Le-Ma 3])

- Case $p = 2$ and $m = 5$ (i.e. curves with genus 2 over a 2-adic field $\subset \mathbb{Q}_2^{\mathrm{tame}}$).
- There are 3 types of degeneration for the marked stable model.
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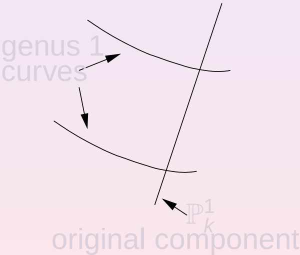

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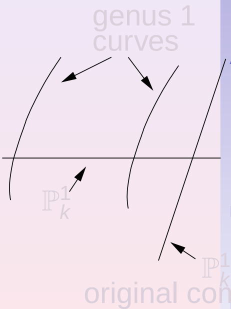
$\mathrm{Gal}(K'/K)_w \hookrightarrow Q_8 \times Q_8$



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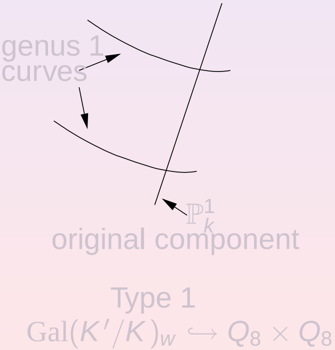

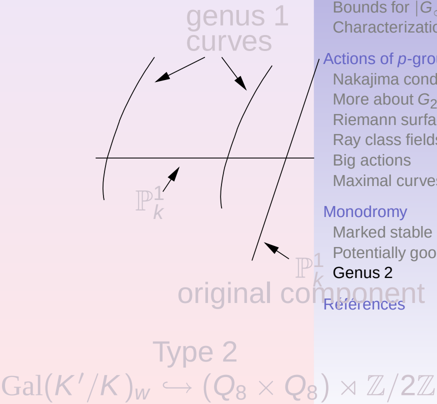

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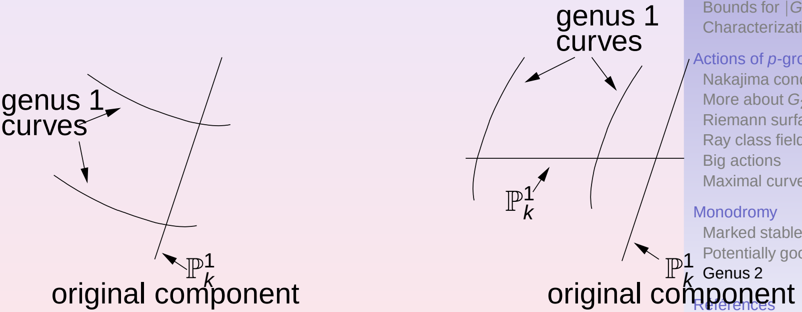

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- R. Auer, *Ray class fields of global function fields with many rational places*, Acta Arith. 95 (2000), no. 2, 97–122.
- M. Conder, *Hurwitz groups: a brief survey*, Bull. Amer. Math. Soc. (N.S.) 23 (1990), no. 2, 359–370.
- N. Elkies, *The Klein quartic in number theory*, The eightfold way, 51–101, Math. Sci. Res. Inst. Publ., 35, Cambridge Univ. Press, Cambridge, 1999.
- K. Lauter, *A formula for constructing curves over finite fields with many rational points*, J. Number Theory 74 (1999), no. 1, 56–72.
- C. Lehr, M. Matignon, *Automorphism groups for p-cyclic covers of the affine line*, Compos. Math. 141 (2005), no. 5, 1213–1237.

- C. Lehr, M. Matignon,*Automorphisms of curves and stable reduction*, in Problems from the workshop on "Automorphisms of Curves" (Leiden, August, 2004), edited by G. Cornelissen and F.Oort, Rend. Sem. Math. Univ. Padova. Vol. 113 (2005), 151-158.
- C. Lehr, M. Matignon, *Wild monodromy and automorphisms of curves* , Duke math. J. à paraître.
- C. Lehr, M. Matignon, *Curves with a big p-group action*, En préparation.
- A. M. Macbeath, *On a theorem of Hurwitz*, Proc. Glasgow Math. Assoc. 5 1961 90–96 (1961).
- A. M. Macbeath, *On a curve of genus* 7, Proc. London Math. Soc. (3) 15 1965 527–542.

M. Marshall, *Ramification groups of abelian local field extensions*, Canad. J. Math. 23 (1971) 271–281.

S. Nakajima, *p-ranks and automorphism groups of algebraic curves*, Trans. Amer. Math. Soc. 303, 595-607 (1987).

M. Raynaud, *p-groupes et réduction semi-stable des courbes*, The Grothendieck Festschrift, Vol.3, Basel-Boston-Berlin: Birkhäuser (1990).

B. Singh, *On the group of automorphisms of function field of genus at least two*, J. Pure Appl. Algebra 4 (1974), 205–229.

H. Stichtenoth, *Über die Automorphismengruppe eines algebraischen Funktionenkörpers von Primzahlcharakteristik. I, II.* Arch. Math. 24 (1973) 527–544 , 615–631.

M. Suzuki, *Group theory II*, Grundlehren der Mathematischen Wissenschaften 248