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Note on the determination of the ignition point in forest fires propagation using a control algorithm

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SUMMARY

This paper is devoted to the determination of the origin point in forest fires propagation using a control algorithm. The forest fires propagation are mathematically modelled starting from a reaction diffusion model. A volume of fluid (V.O.F.) formulation is also used to determine the fraction of the area which is burnt. After having developed the objective functional and its derivative, results from an optimization process based on the simplex method is presented. It is shown that the ignition point and the final time of the fire propagation are precisely recovered, even for a realistic, non-horizontal, terrain. Copyright © 2007 John Wiley & Sons, Ltd.

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KEY WORDS: forest fire; reaction diffusion model; optimization

1. INTRODUCTION

The simulation of forest fire propagation has several purposes. The prevision of the fire front can help fire fighters to optimize the distribution of fire fighting means, which supposes real time simulation. Another application of simulation is fire prevention. By using terrain data, computer models of propagation can provide information on dangerous areas. The possibility for such models to take into account some aspects of the fire fighting means, such as chemical retardants, is highly desirable as well. In this paper we are concerned with applications dedicated to the analysis of the fire departure. Very often the fire is detected after a while, the exact position and time of the ignition are not known and the determination of the position would be valuable information

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for fire men who are in charge of the expertise of managing fire. The aim of this paper is to address the question of the determination of these initial conditions. The model of propagation, or direct model, considered is a simplified version of the reaction diffusion model proposed in Reference [1]. It is a two dimensional model set on the surface of propagation with a non-local heat source term modelling the radiative transfer. The fire front is given by an iso line $T = cte$ of the temperature field. We have introduced a volume of fluid (V.O.F.) like formulation using a function which represents the burnt area density. This is quite a different strategy than the one used by Ferragut *et al.* [2] and Asensio *et al.* [3] where the fire boundary is obtained by a multivalued operator. The advantage of a V.O.F. formulation is that the determination of the critical conditions is formulated as an optimal control problem set with the burnt area density.

The paper is organized as follows: Section 2 is devoted to the presentation of the considered reaction diffusion model, and of the V.O.F. formulation. In Section 3 the objective function is introduced and the differential of the functional is computed. Section 4 is devoted to numerical applications.

2. DIFFUSION REACTION PROPAGATION MODELS

2.1. General description of the model

The model considered is deduced from the balance of energy and the balance of mass for the solid fuel:

$$(1-\Phi)\rho(C_\mathrm{s}+H_\mathrm{u}C_\mathrm{l})\frac{\partial T}{\partial t} = \nabla\cdot(\lambda\nabla T) + h(T_\mathrm{f}-T) + (1-\Phi)\rho\frac{\partial H_\mathrm{u}}{\partial t}L_\mathrm{ev}\delta_{T=T_\mathrm{ev}} + M_\mathrm{r} \tag{1}$$

In this relation T, ρ and Φ represent the temperature, the density of wood and the porosity of the vegetation, C_s and C_l stand for the heat capacity of the dried wood and of water, H_u is the humidity, T_f is the temperature of the ambient gas. M_r is the radiative flux coming from flames. This model corresponds to a case where the total energy received by the fuel is used for evaporating the water during the process of drying. The model uses the assumptions that drying and pyrolysis are not concomitant, drying occurs at constant temperature, constant volume during drying and pyrolysis processes, and the char oxidation is neglected. If the water is free (not inside the vegetal) one has to change the value of the 'latent' heat L_ev. In the zone where pyrolysis occurs, an equation modelling the kinetic of decomposition must be considered:

$$\frac{\partial\rho}{\partial t} = -\rho f(T) \tag{2}$$

Indeed the preceding model can be summed up as follows:

(i) In the zone before the evaporation front, denoted by zone I, such that $T < T_\mathrm{ev}$ and $\rho > \rho_\mathrm{ext}$

$$(1-\Phi)\rho(C_\mathrm{s}+H_{\mathrm{u}0}C_\mathrm{l})\frac{\partial T}{\partial t} = \nabla\cdot(\lambda\nabla T) + M_\mathrm{r} + h(T_\mathrm{f}-T) \tag{3}$$

where C_s is the heat capacity of the solid constituent of vegetation, C_l is the heat capacity of the water, $H_{\mathrm{u}0}$ is the initial humidity, h is the heat loss coefficient, T_f is the temperature of the gaseous phase.

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(ii) In the evaporation zone, denoted by zone II, such that $T = T_{\mathrm{ev}}$, $H_{\mathrm{u}} > 0$ and $\rho \geqslant \rho_{\mathrm{ext}}$

$$-(1-\Phi)\rho L_{\mathrm{ev}} \frac{\partial H_{\mathrm{u}}}{\partial t} = M_{\mathrm{r}} - h(T - T_{\mathrm{f}}) \quad (4)$$

L_{ev} is an evaporation latent heat and ρ_{ext} denotes the extinction density of wood.

(iii) In the intermediary zone between the evaporation zone and the burning zone, denoted by zone III, such that $T_{\mathrm{ev}} < T < T_{\mathrm{i}}$, $H_{\mathrm{u}} = 0$ and $\rho \geqslant \rho_{\mathrm{ext}}$

$$(1-\Phi)\rho C_{\mathrm{s}} \frac{\partial T}{\partial t} = \nabla \cdot (\lambda \nabla T) + M_{\mathrm{r}} - h(T - T_{\mathrm{f}}) \quad (5)$$

T_{i} is the ignition temperature.

(iv) In the burning zone, denoted by zone IV, such that $T \geqslant T_{\mathrm{i}}$, $H_{\mathrm{u}} = 0$ and $\rho \geqslant \rho_{\mathrm{ext}}$

$$(1-\Phi)\rho C_{\mathrm{s}} \frac{\partial T}{\partial t} = \nabla \cdot (\lambda \nabla T) + M_{\mathrm{r}} - h(T - T_{\mathrm{f}}) \quad (6)$$

The variation of mass due to chemical reactions is

$$\frac{\partial \rho}{\partial t} = -\rho A \exp(-E/RT) \quad (7)$$

where A is a constant and E is the activation energy of the pyrolysis.

(v) The burnt zone, denoted by zone V, such that $\rho = \rho_{\mathrm{ext}}$

$$(1-\Phi)\rho C_{\mathrm{s}} \frac{\partial T}{\partial t} = \nabla \cdot (\lambda \nabla T) M_{\mathrm{r}} - h(T - T_{\mathrm{f}}) \quad (8)$$

All the preceding described regions are illustrated in Figure 1.

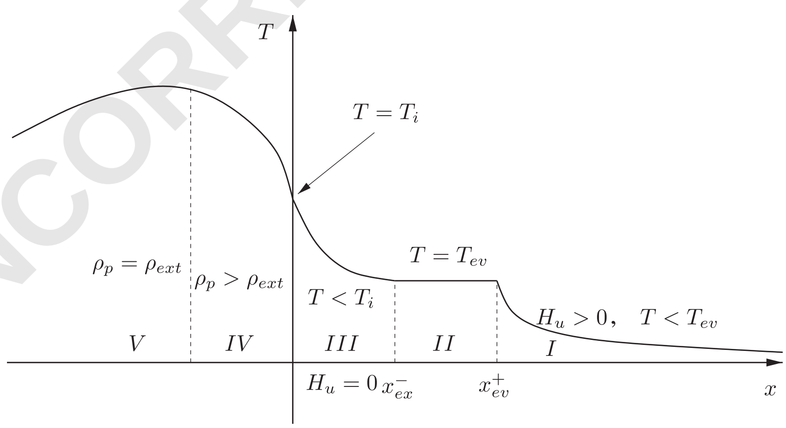

Figure 1. Different zones related to the spreading in a one dimensional propagation, the evaporation zone is the interval $]x_{\mathrm{ev}}^-, x_{\mathrm{ev}}^+[$.

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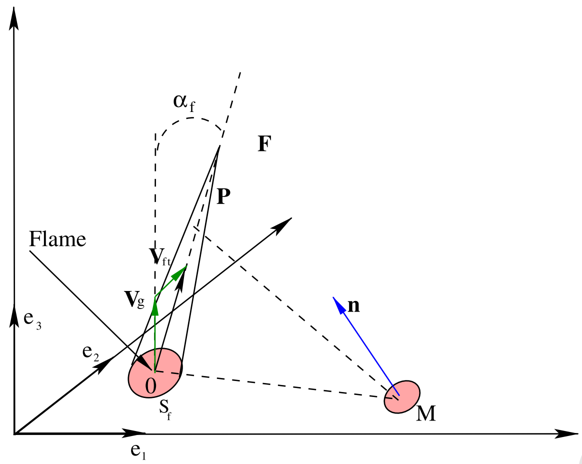

Figure 2. Radiation of the flame.

2.2. *Flame model*

The vegetation is supposed to be thin and set on a plane S_f. The flame is supposed to be at constant known temperature T_f and each flame element is supposed to be directed by a unit vector F parallel to the velocity of the gas V_f, the emitting point is denoted by P and the receiving point by M, O is the flame foot (cf. Figure 2).

The global unit vectors of the global co-ordinate system are denoted by (e_1, e_2, e_3), e_3 being the vertical direction. The vector n is the unit normal to the upper plane (i.e. unit normal to the receiving surface) of the vegetation at point M. The angle between F and the vertical is denoted by $\alpha_\mathrm{f} = (e_3, F)$. The flame elements are supposed to have a length l_f. If the fire front is supposed to be thin, one can show that the radiative heat flux is given by convolution integral calculated on the burning zone denoted by S_f:

$$\begin{aligned} M_\mathrm{r} &= \int_{S_\mathrm{f}} \phi(y) G(x-y)\,\mathrm{d}y_1\,\mathrm{d}y_2 \\ &= K_\mathrm{f} \frac{BT^4}{\pi} \int_{S_\mathrm{f}} \frac{(\mathbf{F}(1-\cos\theta_\mathrm{f}) - \mathbf{w}(\cos\beta - \cos(\beta+\theta_\mathrm{f})))\cdot \mathbf{n}}{r \sin^2 \beta}\,\mathrm{d}x_1\,\mathrm{d}x_2 \end{aligned} \tag{9}$$

with, cf. Appendix A for notation and derivation, r being the distance between the emitting and the receiving points, and K_f the absorption coefficient of the flame.

The velocity of the gas is the sum of the vertical velocity of the gases and of the wind

$$\mathbf{V}_\mathrm{f} = \mathbf{v}_\mathrm{g} + \mathbf{V} \tag{10}$$

where $\mathbf{v}_\mathrm{g} = \sqrt{g h_\mathrm{f}}\mathbf{e}_3$ is the vertical flame gas velocity, h_f being the flame height, and $\mathbf{V}$ is the wind velocity.

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2.3. V.O.F. formulation

In fact the propagation of fire in the preceding modelling is a free boundary problem. Let us derive a V.O.F. formulation [4]. For the sake of simplicity let us assume that there is no humidity that is $H_u = 0$. Let $\chi(\mathbf{x}, t)$ be the characteristic function of the burning zone, i.e. $\chi(\mathbf{x}, t) = 1$ if the point x lies in the burning zone at time t and $\chi(\mathbf{x}, t) = 0$ elsewhere. The characteristic function, considered as a distribution, satisfies the equation

$$\frac{\partial \chi}{\partial t} + \nabla \chi \cdot \boldsymbol{\omega} = 0 \tag{11}$$

In relation (11) $\mathbf{w}$ is the normal velocity of the fire front; it is equal to

$$\mathbf{w} = - \left. \frac{\frac{\partial T}{\partial t}}{|\nabla T|^2} \nabla T \right|_{T = T_i} \tag{12}$$

We can consider now a mollifier $m_h(\mathbf{x})$, i.e. a function such that $m_h(\mathbf{x}) > 0$, $\int_{R^2} m_h(\mathbf{x})\,d\mathbf{x} = 1$ and the function tends to a Dirac distribution when $h \to 0$. This function can be chosen as smooth as desired, so that the function defined by the convolution

$$\alpha(\mathbf{x}, t) = (\chi * m_h)(\mathbf{x}, t) = \int_{R^2} \chi(\mathbf{y}, t) m_h(\mathbf{x} - \mathbf{y})\,d\mathbf{y} \tag{13}$$

is a regular approximation of the characteristic function χ. Now an approximation of the system of propagation is

$$(1 - \Phi)\rho C_s \frac{\partial T}{\partial t} = \nabla \cdot (\lambda(\alpha)\nabla T) + M_r(\alpha) - h(T - T_f) \tag{14}$$

$$\frac{\partial \alpha}{\partial t} + \nabla \alpha \cdot \mathbf{w} = 0 \tag{15}$$

$$\frac{\partial \rho}{\partial t} = -\rho A \exp(-E/RT) \tag{16}$$

and now the radiative heat is given by

$$M_r = \int_{R^2} \alpha(y, t)\phi(y) G(x - y)\,dy_1\,dy_2 \tag{17}$$

The velocity w is an extension of the front velocity given by

$$\mathbf{w} = - \frac{\frac{\partial T}{\partial t}}{|\nabla T|^2} \nabla T \tag{18}$$

The system (13)–(18) is now set on the whole plane R^2. The study of the numerical algorithm for solving this system is not addressed in this paper.

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3. THE CONTROL ALGORITHM FOR THE DETERMINATION OF THE IGNITION POINT

3.1. Definition of the objective function and computation of its differential

Let us consider now at time t_f a burnt area whose characteristic function is denoted by χ_f, to which a function α_f corresponds by convolution. In fact, we are interested in finding the position (x_0, y_0) and the time t_0 of ignition. Let us add to the system (14)–(18) the initial conditions

$$\chi(x, y, t_0) = \delta_{x=x_0,\, y=y_0} \tag{19}$$

$$T(x, y, t_0) = T_i \delta_{x=x_0,\, y=y_0} \tag{20}$$

or the related relations written with the regularized function α.

It is now natural to set the problem as an optimal control problem.

$$\text{Find } (x_0, y_0, t_0) \text{ such that } J = \int_{R^2} |\chi(x, y, t_f) - \chi_f(x, y)|^2 \, dx \, dy \text{ is minimum} \tag{21}$$

The numerical calculation of the integral J in problem (21) will be replaced by the approximation

$$J_a = \int_{R^2} -\alpha(x, y, t_f)\alpha_f(x, y)|^2 \, dx \, dy \tag{22}$$

In fact J is an integral calculated on the domain $\Omega - \Omega_f$, Ω being the domain whose characteristic function is χ and can be written as

$$J = \int_{\Omega - \Omega_f} dx \, dy \tag{23}$$

In practice, the constrained optimization problem (21) can be solved using two different classes of algorithms. The first class is composed of the stochastic algorithms like the genetic ones. These kinds of algorithms permit the finding of the global optimizer of an objective function but for a prohibitive computational time in engineering applications. The second class, composed mainly of the gradient-based methods, seems to be more efficient but is not guaranteed to converge towards the global optimizer. Under these remarks it is thus clear that a balance between the computational time and the performance of the solution has to be made. In order to reduce the computational costs involved during stochastic process Amirjanov [5] has developed a new class of genetic algorithm where the range of the exploration region is re-adapted for each generation. Stochastic procedures are also often used with interpolation methods [6, 7]. In this paper we choose to solve the constrained optimization problem (21) with two deterministic methods. The first one is the Polack–Ribière conjugate gradient algorithm, which requires the determination of the differential of J, coupling with a backtracking Armijo line search. The second one is the nonlinear Nelder Mead simplex method [8] which does not require the differentiability of the objective functional. Both algorithms have been used numerically, and for the sake of completeness we give the differential of the objective functional with respect to x_0; analog results would be obtained for the other components. The differential of J can be written as

$$\frac{\partial J}{\partial x_0} = \int_{R^2} 2(\chi(x, y, t_f) - \chi_f(x, y)) \frac{\partial \chi}{\partial x_0} \, dx \, dy \tag{24}$$

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The term $\partial\chi/\partial x_0$ being solution of the system of equations

$$(1-\Phi)\left(\rho C_{\mathrm{s}}\frac{\partial T'}{\partial t}+\rho' C_{\mathrm{s}}\frac{\partial T}{\partial t}\right)=\nabla\cdot(\lambda(\chi')\nabla T)+\nabla\cdot(\lambda(\chi)\nabla T')+M_{\mathrm{r}}(\chi')-hT' \tag{25}$$

$$\frac{\partial\chi'}{\partial t}+\nabla\chi'\cdot\mathbf{w}+\nabla\chi\cdot\mathbf{w}'=0 \tag{26}$$

$$\frac{\partial\rho'}{\partial t}=-\chi'\rho A\exp(-E/RT)-\chi\rho' A\exp(-E/RT)-\chi\rho\frac{ET'}{RT^2}A\exp(-E/RT) \tag{27}$$

$$M_{\mathrm{r}}=\int_{R^2}\chi'(y,t)\phi(y)G(x-y)\,\mathrm{d}y_1\,\mathrm{d}y_2 \tag{28}$$

The notation T' stands for $\partial T/\partial x_0$, and the initial conditions are

$$T'(x,y,t_0)=T_{\mathrm{i}}\delta'_{x=x_0,\,y=y_0} \tag{29}$$

$$\chi'(x,y,t_0)=\delta'_{x=x_0,\,y=y_0} \tag{30}$$

The distribution $\delta'_{x=x_0,\,y=y_0}$ is defined by

$$\langle\varphi,\delta'_{x=x_0,\,y=y_0}\rangle=-\frac{\partial\varphi}{\partial x}(x_0,y_0) \tag{31}$$

for any function with compact support in R^2.

In fact the terminal position of the fire front will never be known exactly; we will consider perturbations $\delta\chi_{\mathrm{f}}$ of the boundary. Let us consider virtual motion

$$x\mapsto x+\varepsilon\tau(x) \tag{32}$$

in such a way that every point x is moved up to first order in ε. Let us define the translated characteristic function as

$$\overline{\chi}_{\mathrm{f}}(\varepsilon)=\chi_{\mathrm{f}}(x+\varepsilon\tau(x)) \tag{33}$$

Then the variations of the characteristic function are defined by

$$\delta\chi_{\mathrm{f}}=\varepsilon\frac{\partial\overline{\chi}_{\mathrm{f}}}{\partial\varepsilon}(\varepsilon=0)=\varepsilon\nabla\chi_{\mathrm{f}}\cdot\tau=-\varepsilon\delta_{\partial\Omega_{\mathrm{f}}}n\cdot\tau \tag{34}$$

In relation (34), $\delta_{\partial\Omega_{\mathrm{f}}}$ stands for the Dirac distribution on $\partial\Omega_{\mathrm{f}}$, n is the unit outward normal. Then we can compute the variation, or sensitivity of the objective functional:

$$\delta J=J(\chi_{\mathrm{f}}+\delta\chi_{\mathrm{f}})-J(\chi_{\mathrm{f}})=-2\int_{\Omega_{\mathrm{f}}}(\chi-\chi_{\mathrm{f}})\delta\chi_{\mathrm{f}}\,\mathrm{d}x\,\mathrm{d}y+\int_{\Omega_{\mathrm{f}}}(\delta\chi_{\mathrm{f}})^2\,\mathrm{d}x\,\mathrm{d}y \tag{35}$$

The computation of δJ must be done for $\chi=\chi_{\mathrm{f}}$ then

$$\delta J=\int_{\Omega_{\mathrm{f}}}(\delta\chi_{\mathrm{f}})^2\,\mathrm{d}x\,\mathrm{d}y=\varepsilon^2\int_{\partial\Omega_{\mathrm{f}}}\tau^2\,\mathrm{d}s \tag{36}$$

where ds denotes the curvilinear co-ordinate along the line boundary $\partial\Omega_{\mathrm{f}}$.

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In Equation (36) we have set $\tau = \boldsymbol{\tau} \cdot \boldsymbol{n}$. As a function defined on the boundary $\partial\Omega_{\mathrm{f}}$ of the final domain, τ is a periodic function of the arc length s and can be developed in Fourier series:

$$\tau = \sum_n \tau_n \exp(-2\mathrm{i}\pi n s/L) \tag{37}$$

where L is the total length of $\partial\Omega_{\mathrm{f}}$. With these notations, the sensitivity of the functional is

$$\delta J = \varepsilon^2 \sum_n \tau_n^2 \tag{38}$$

4. SOME COMPUTATIONAL APPLICATION

In order to test the algorithm we have first considered a propagation on a horizontal terrain with a uniform density of vegetation, the parameters of the propagation model are the one considered in Table I. Note that we have not taken into account for this simulation the diffusivity term $\nabla \cdot (\lambda \nabla T)$ in Equation (1) (and in the other ones derived from this one) because the value of the parameter λ is not yet physically well defined.

The ignition point is arbitrarily chosen such that $x_0 = y_0 = 1000$ m. After $t_{\mathrm{f}} = 120$ mn of propagation the fire front being a circle has been perturbed by a sinus, that is

$$\delta\chi_{\mathrm{f}} = -\varepsilon\delta_{\partial\Omega_{\mathrm{f}}} \boldsymbol{n} \cdot \boldsymbol{\tau} = a \sin(\omega s \delta_{\partial\Omega_{\mathrm{f}}}) \tag{39}$$

so that the equation for the new boundary is

$$\rho = r + a \sin(2\pi n r\theta/2\pi r) = r + a \sin(n\theta) \tag{40}$$

in polar co-ordinate. This new curve is considered as the noisy perturbated final fire front, see Figure 3.

As mentioned before, two optimization algorithms have been used, Polack–Ribière conjugate gradient and Nelder Mead simplex, see Reference [8]. The most efficient has revealed to be the simplex and the results described here are the ones obtained in this way.

Figure 5 represents the evolution of the calculated fire ignition point position at each iteration *versus* the optimization iterations numbers. It is noticeable that after 30 iterations, the value of

Table I. Parameter values of the propagation model.

$C_{\mathrm{s}} = 2\,400\,\mathrm{J\,kg^{-1}\,K^{-1}}$
$\delta = 1\,\mathrm{m}$
$h = 20\,\mathrm{J\,m^{-2}\,s^{-1}\,K^{-1}}$
$Ta = 300\,\mathrm{K}$
$h_{\mathrm{f}} = 2\,\mathrm{m}$
$\Phi = 0.9$
$C_{\mathrm{l}} = 4\,180\,\mathrm{J\,kg^{-1}\,K^{-1}}$
$K_{\mathrm{f}} = 0.2\,\mathrm{m^{-1}}$
$L_{\mathrm{ev}} = 2.250 \times 10^6\,\mathrm{J\,kg^{-1}}$
$T_{\mathrm{ev}} = 373\,\mathrm{K}$
$T_{\mathrm{f}} = 1200\,\mathrm{K}$
$\rho_{\mathrm{ext}} = 3\,\mathrm{kg\,m^{-3}}$

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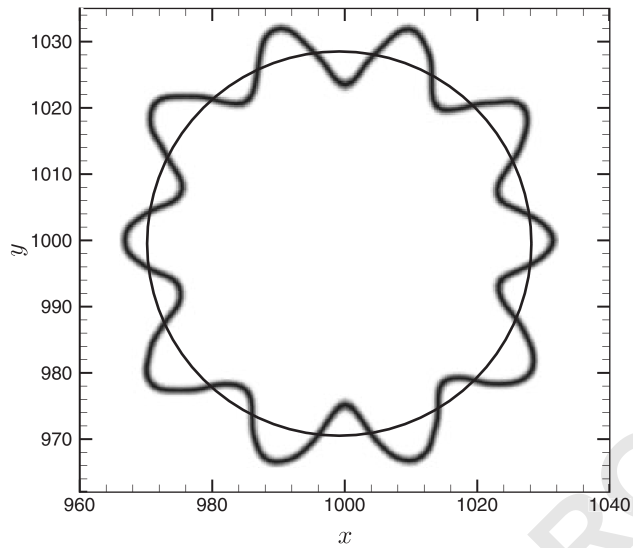

Figure 3. Real and perturbated fire front.

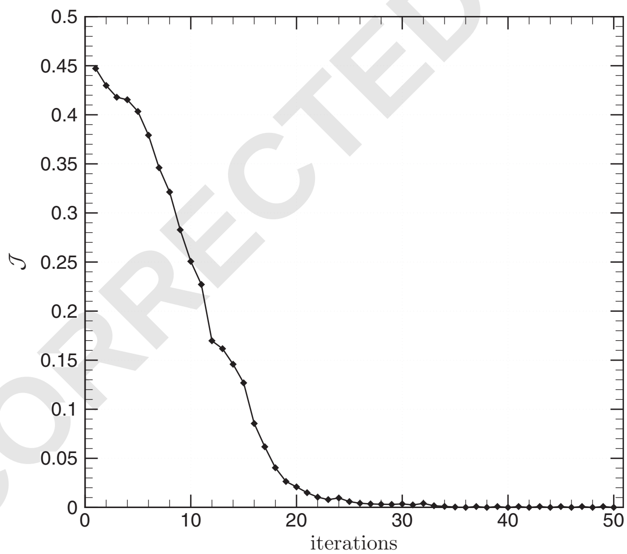

Figure 4. Variation of the functional as a function of the number of iterations.

the objective function stops to decrease significantly (see Figure 4), the real ignition point, i.e. $x_0 = 1000$ m and $y_0 = 1000$ m, is precisely recovered.

After the same optimization iteration numbers, the final propagation time $t_f = 120$ mn is also precisely recovered as we can see in Figure 6.

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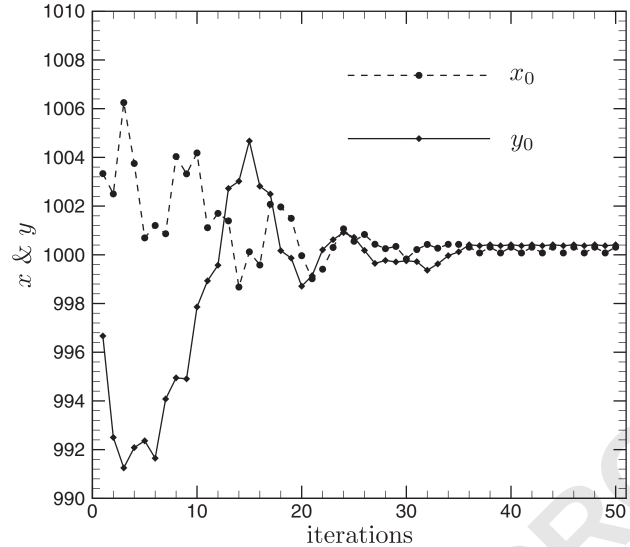

Figure 5. Variation of initial positions *versus* the number of iterations.

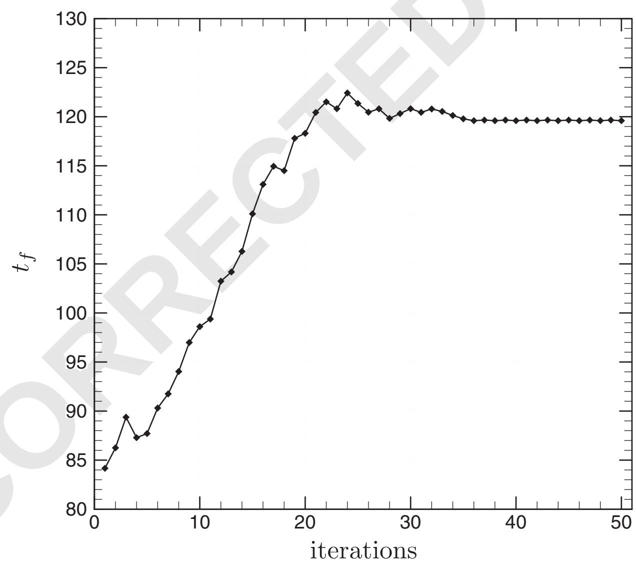

Figure 6. Variation of duration time *versus* the number of iterations.

In order to test the efficiency of this study we have also taken into consideration a much more complicated case. We have thus considered a propagation on a non-horizontal terrain, with a uniform density of vegetation and with the same parameters as in the previous case. The topology of the terrain under consideration is composed of two distinct bumps as one can show in Figure 8.

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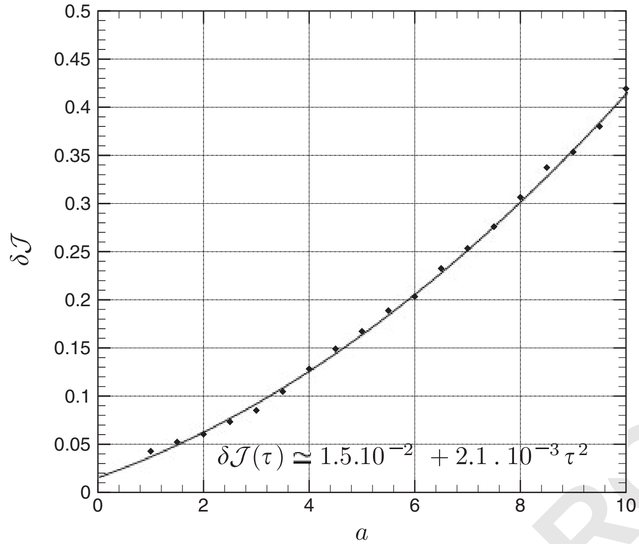

Figure 7. Sensibility of the objective function.

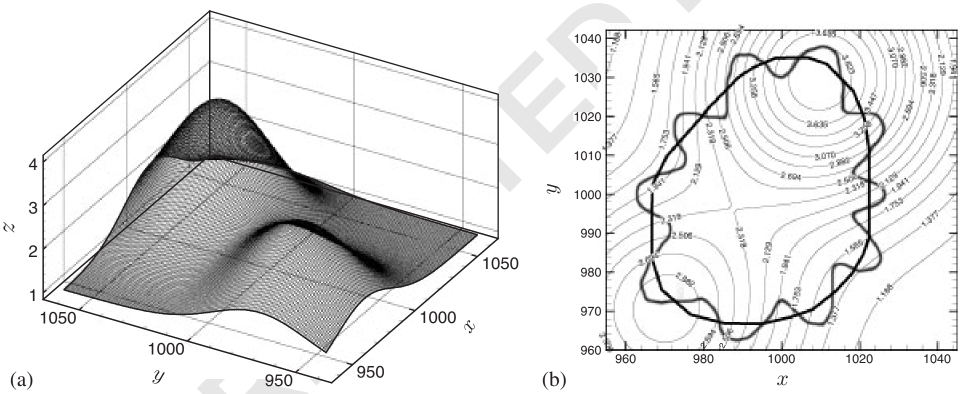

Figure 8. Topology of the non-uniform terrain (a) 3D visualisation (b) visualisation.

The z-co-ordinates of the terrain are given by

$$z(x, y) = z_0 + h_{b1} \exp \frac{-(x - x_{b1})^2}{A} \exp \frac{-(y - y_{b1})^2}{A} + h_{b2} \exp \frac{-(x - x_{b2})^2}{A} \exp \frac{-(y - y_{b2})^2}{A} \tag{41}$$

where $z_0 = 1$ m, the co-ordinate of the centre of the first and second bumps are $(x_{b1}, y_{b1}) = (1010\,\text{m}, 1030\,\text{m})$ and $(x_{b2}, y_{b2}) = (970\,\text{m}, 970\,\text{m})$, respectively, the relative maximal height of the first and second bumps are $h_{b1} = 3$ m and $h_{b2} = 2$ m, respectively, and the value of the smooth factor surface A is $1000\,\text{m}^2$.

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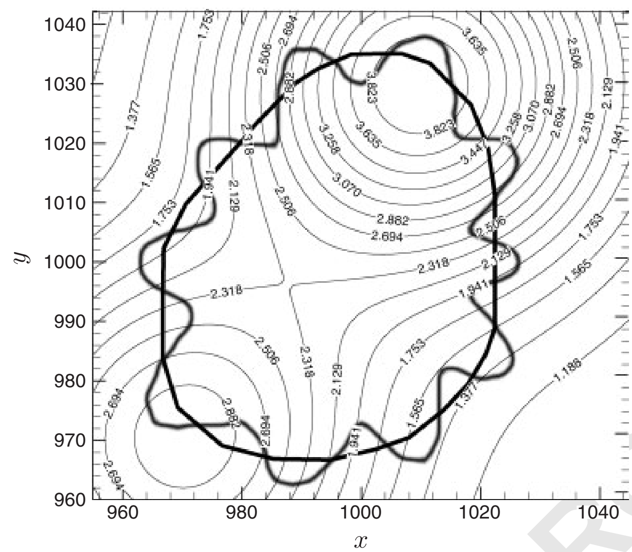

Figure 9. Real and perturbated fire front. Non-uniform terrain.

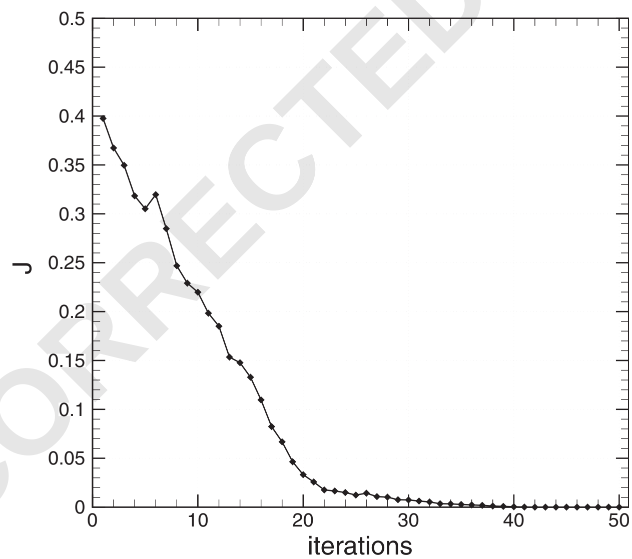

Figure 10. The functional as a function of the number of iterations. Non-uniform terrain.

The ignition point is once again arbitrary taken to be $x_0 = y_0 = 1000\,\mathrm{m}$. After $t_\mathrm{f} = 120\,\mathrm{mn}$ of propagation the fire front, which is not a circle now, has been perturbed by a sinus too (see Figure 9).

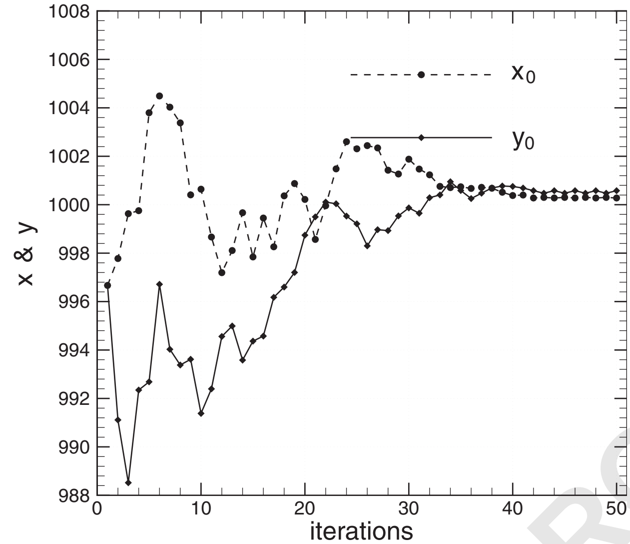

Figure 11. Variation of initial positions *versus* the number of iterations. Non-uniform terrain.

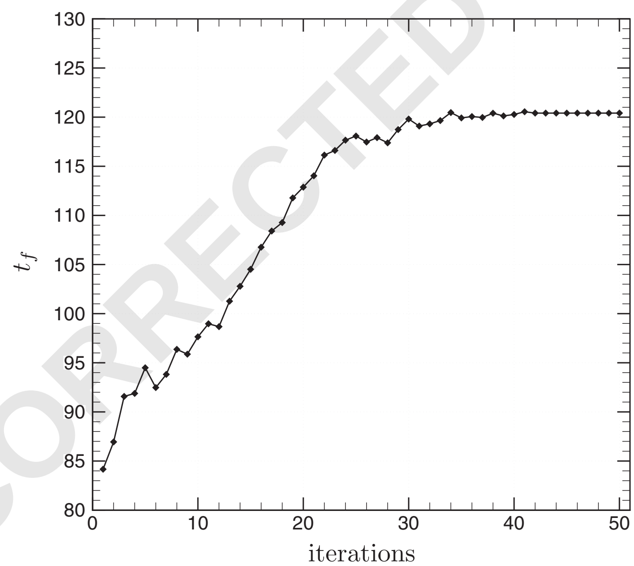

Figure 12. Variation of final time *versus* the number of iterations. Non-uniform terrain.

Once again, after 30 iterations the value of the objective functional tends to zero (see Figure 10). The ignition point $x_0 = y_0 = 1000$ m and the final time $t_f = 120$ mn are also precisely recovered (see Figures 11 and 12, respectively).

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The sensitivities of the objective function are now studied. Several numerical optimizations have been done for different values of the parameters a located between 1 and 10 by steps equal to 1, for a fixed value $n = 20$, see Equation (40) for the signification of a and n. Each numerical optimization gave rise to an optimal value $J_{\min}$ of the objective function which is nothing else than δJ because $J(\chi_{\mathrm{f}}) = 0$. These values are plotted in Figure 7. It is noticeable as foreseen in relations (37) and (38) that the optimal value $J_{\min}$ varies approximately as the square of the parameters a. Indeed, the more the fire front is perturbated the more it is difficult to recover the ignition point.

These simple numerical experiments show that it is possible to recover the initial point and time of departure of the fire, in a deterministic way, using an optimal control formulation and a simplex optimization algorithm. However, the existence and uniqueness of the optimal solution of such an optimization problem is still an open question. Numerically, it has to be precisely that the same solutions are obtained in a stochastic way using a standard genetic algorithm. It is shown in practice that these kinds of algorithms almost give the optimal solution, but at high numerical costs.

5. CONCLUSION

This paper is dedicated to the finding of the ignition point in forest fires propagation. The problem has been set as an optimal control problem. The diffusion has not been taken into account for the sake of simplicity and because diffusivity has been obtained after a homogenization process and its value are not clearly determined. This will be the purpose of another paper. The objective functional and its derivatives *versus* each control parameters were derived. This gradient has been used in a conjugate gradient optimization method, but the simplest and effective optimization method is the simplex one. Using this optimization method the ignition point and the final time of forest fires propagation were precisely recovered even for a realistic non-horizontal terrain where only noisy measurement of the fire line is available. This will be extended to a diffusive model and with a fire front randomly perturbated. These results could be very helpful for firemen who are in charge of expert management of the fire.

APPENDIX A: DERIVATION OF THE RELATION (9)

Let us now derive relation (9).

We will assume that the flame and the vegetation are grey medium with constant absorption coefficients K_{f}, K_{v}. Then if the temperature T_{f} of the flame is constant the integration of the radiative transfer equation gives for the intensity

$$i(s) = \frac{BT_{\mathrm{f}}^4}{\pi}(1 - \mathrm{e}^{-K_{\mathrm{f}}(s_2 - s_1)})\mathrm{e}^{-K_{\mathrm{v}}(s - s_3)} + K_v \int_{s_3}^{s} i_b(\bar{s})\mathrm{e}^{-K_{\mathrm{v}}(s - \bar{s})}\,\mathrm{d}\bar{s} \tag{A1}$$

If the flame is assumed thin $(1 - \mathrm{e}^{-K_{\mathrm{f}}(s_2 - s_1)}) = K_{\mathrm{f}}(s_2 - s_1) = K_{\mathrm{f}} \int_{s_1}^{s_2} \mathrm{d}\bar{s}$ and the preceding relation becomes

$$i(s) = K_{\mathrm{f}} \frac{BT_{\mathrm{f}}^4}{\pi} \mathrm{e}^{-K_{\mathrm{v}}(s - s_3)} \int_{s_1}^{s_2} \mathrm{d}\bar{s} + K_{\mathrm{v}} \int_{s_3}^{s} i_b(\bar{s})\mathrm{e}^{-K_{\mathrm{v}}(s - \bar{s})}\,\mathrm{d}\bar{s} \tag{A2}$$

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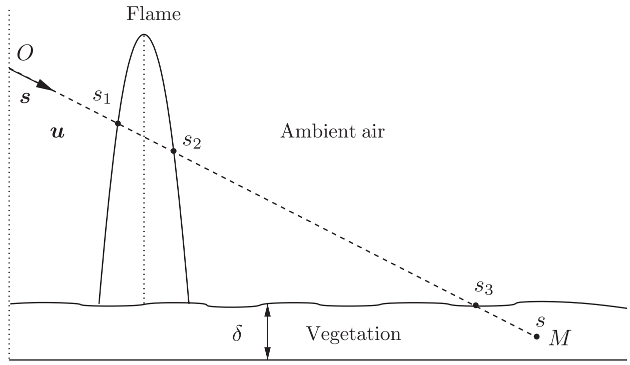

Figure A1. Configuration of the flame.

Then the radiative flux density received by a surface with normal $\mathbf{n}_i$ is

$$\mathbf{q}_{\mathrm{r}}(M)\cdot\mathbf{n}_i = K_{\mathrm{f}}\frac{BT_{\mathrm{f}}^4}{\pi}\int_{\Omega_{\mathrm{f}}}\frac{\mathrm{e}^{-K_{\mathrm{v}}aM}}{PM^2}\mathbf{u}\cdot\mathbf{n}_i\,\mathrm{d}\Omega(P) + K_{\mathrm{v}}\frac{BT_{\mathrm{v}}^4}{\pi}\int_{\Omega_{\mathrm{v}}}\frac{\mathrm{e}^{-K_{\mathrm{v}}AM}}{AM^2}\mathbf{u}\cdot\mathbf{n}_i\,\mathrm{d}\Omega(P) \tag{A3}$$

where a and A denote points belonging to the vector $\mathbf{u}=\mathbf{OM}$ (see Figure A1) which are, respectively, located inside the vegetation and at the top of the vegetation where Ω_{f} is the domain occupied by the flame and Ω_{v} is the domain occupied by the vegetation. In the limit $\delta\to 0$ the right-hand side of (A2) reduces to

$$\mathbf{q}_{\mathrm{r}}(M)\cdot\mathbf{n} = K_{\mathrm{f}}\frac{BT_{\mathrm{f}}^4}{\pi}\int_{\Omega_{\mathrm{f}}}\frac{1}{PM^2}\mathbf{u}\cdot\mathbf{n}\,\mathrm{d}\Omega(P) \tag{A4}$$

This triple integral can be reduced to a double integral. Let us consider that each element of flames is directed by a unit vector $\mathbf{F}$, $\mathbf{n}$ is the unit vector normal to the plane Π_{v} which is the top of vegetation at the receiving point M, $\mathbf{f}$ is the unit vector representing the direction of the orthogonal projection of the flame $\mathbf{F}$ on the plane, see Figure 2.

If we consider an absolute co-ordinate system $(O,\mathbf{e}_1,\mathbf{e}_2,\mathbf{e}_3)$ (not drawn on figure above), we define the following angles:

$$(\mathbf{e}_3,\mathbf{F}) = \alpha_{\mathrm{f}}, \quad (\mathbf{F},\mathbf{OM}) = \beta \tag{A5}$$

The emitting point on the flame is the point P, the point O is the flame foot , and we consider the radial co-ordinate r, such that

$$\|\mathbf{PM}\| = \rho, \quad \mathbf{OP} = \xi\mathbf{F} \quad \text{and} \quad \mathbf{OM} = r\mathbf{w} \tag{A6}$$

Then $\mathbf{PM}=\mathbf{OM}-\mathbf{OP}=r\mathbf{w}-\xi\mathbf{F}$, and $\mathbf{u}\cdot\mathbf{n}=(1/\rho)(r\mathbf{w}\cdot\mathbf{n}-\xi\mathbf{F}\cdot\mathbf{n})$.

In the triple integral of the right-hand side of (A4) we integrate first along the flame, with the previous notations, we obtain

$$M_{\mathrm{r}} = -\mathbf{q}_{\mathrm{r}}(M)\cdot\mathbf{n} = -K_{\mathrm{f}}\frac{BT_{\mathrm{f}}^4}{\pi}\int_{S_{\mathrm{f}}}\frac{\mathrm{d}x\,\mathrm{d}y}{\cos\alpha_{\mathrm{f}}}\int_0^{l_{\mathrm{f}}}\frac{1}{\rho^3}(r\mathbf{w}\cdot\mathbf{n}-\xi\mathbf{F}\cdot\mathbf{n})\,\mathrm{d}\xi \tag{A7}$$

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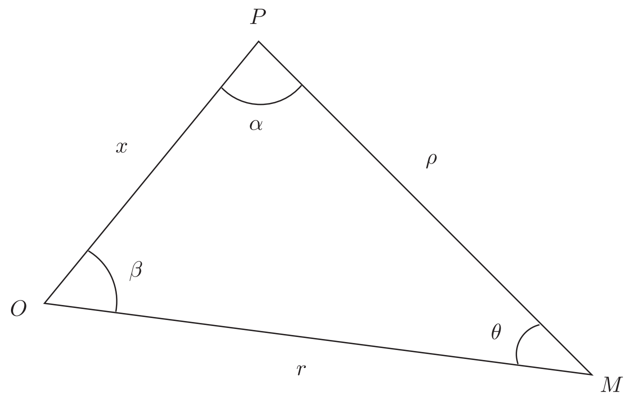

Figure A2. Definition of the different angles in the triangle *OPM*.

In (A7) S_f is the burning surface and l_f the local flame length. The simple integrals $\int_0^{l_\mathrm{f}} \frac{\xi}{\rho^3}\,\mathrm{d}\xi$ and $\int_0^{l_\mathrm{f}} \frac{r}{\rho^3}\,\mathrm{d}\xi$ can be evaluated. Let us consider the triangle *OPM* and the different angles in this triangle cf. Figure A2.

Then we have the following relations:

$$\frac{\rho}{\sin\beta} = \frac{r}{\sin\alpha} = \frac{\xi}{\sin\theta} \quad \text{with } \alpha+\beta+\theta=\pi \tag{A8}$$

Then θ can be used as parameter. The derivation of (A8) gives

$$\mathrm{d}\xi = r\frac{\sin\beta}{\sin^2(\beta+\theta)}\,\mathrm{d}\theta = r\frac{\sin\beta}{\sin^2\alpha}\,\mathrm{d}\theta \tag{A9}$$

Then

$$I_1 = \int_0^{l_\mathrm{f}} \frac{r}{\rho^3}\,\mathrm{d}\xi = \frac{1}{r\sin^2\beta}\int_0^{\theta_{\mathrm{f}_m}} \sin(\beta+\theta)\,\mathrm{d}\theta = \frac{1}{r\sin^2\beta}(\cos\beta - \cos(\beta+\theta_{\mathrm{f}_m})) \tag{A10}$$

$$I_2 = \int_0^{l_\mathrm{f}} \frac{\xi}{\rho^3}\,\mathrm{d}\xi = \frac{1}{r\sin^2\beta}\int_0^{\theta_{\mathrm{f}_m}} \sin\theta\,\mathrm{d}\theta = \frac{1}{r\sin^2\beta}(1-\cos\theta_{\mathrm{f}_m}) \tag{A11}$$

Once (A10) and (A11) are put in the integral one obtains

$$M_\mathrm{r} = K_\mathrm{f}\frac{BT_\mathrm{f}^4}{\pi}\int_{S_\mathrm{f}} \frac{\{\mathbf{F}(1-\cos\theta_{\mathrm{f}_m}) - \mathbf{w}(\cos\beta - \cos(\beta+\theta_{\mathrm{f}_m}))\}\cdot\mathbf{n}}{r\sin^2\beta}\,\mathrm{d}x\,\mathrm{d}y \tag{A12}$$

With θ_{f_m} and β solutions to the equations

$$\cos\beta = \mathbf{F}\cdot\frac{\mathbf{OM}}{r} \quad \text{and} \quad \cot\theta_{\mathrm{f}_m} = \frac{r}{l_\mathrm{f}\sin\beta} - \cot\beta \tag{A13}$$

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APPENDIX B. DESCRIPTION OF THE OPTIMAL CONTROL ALGORITHM

We consider the following optimization problem:

$$\min_{\mathbf{x}} J(x) \tag{B1}$$

with $J : R^n \mapsto R$ is a continuous function. The gradient of the objective function J according to some control parameters $\mathbf{x}$ is denoted by ∇J.

As it was mentioned in Section 1, we use the Polack–Ribière conjugate gradient algorithm coupling with a backtracking Armijo line search.

B.1. Polack–Ribière conjugate gradient algorithm

Choose an initial iterate $\mathbf{x}_0$. Computation of $J_0 = J(\mathbf{x}_0)$ and $\nabla J_0 = \nabla J(\mathbf{x}_0)$. The algorithm is initialized by a steepest descent step, $\mathbf{d}_0 = -\nabla J_0$.

While a stopping criterion is not satisfied, do:

1. Determination of a step α_k (see Section 2). Computation of a new iterate $x_{k+1} = x_k + \alpha_k \mathbf{d}_k$;
2. Evaluation of a new gradient ∇J_{k+1};
3. Construction of a new direction of descent $\mathbf{d}_{k+1} = -\nabla J_{k+1} + \beta_{k+1} \mathbf{d}_k$, with

$$\beta_{k+1} = \frac{\nabla J_{k+1}^T (\nabla J_{k+1} - \nabla J_k)}{\nabla J_k^T \nabla J_k}$$

4. $k = k + 1$

B.2. Backtracking Armijo line search

The Armijo condition writes

$$J(\mathbf{x}_k + \alpha_k \mathbf{d}_k) \leqslant J(\mathbf{x}_k) + \omega_1 \alpha_k \langle \nabla J_k, \mathbf{d}_k \rangle \tag{B2}$$

where ω_1 is a small parameters (usually $\omega_1 = 10^{-4}$).

Initializations: choose a step $\alpha_k^1 > 0$ and a parameter $\tau \in]0, 1[$. $i = 1$.

1. Test: the step α_k^i is accepted if it verifies the Armijo relation (B2)
2. If not:
3. Choose $\alpha_k^{i+1} \in [\tau \alpha_k^i, (1-\tau)\alpha_k^i]$
4. $i = i + 1$ and $\alpha_k = \alpha_k^i$. Return to 1.

The parameter τ is usually taken to be equal to 10^{-2}.

Usually, only 2 or 3 evaluations of the objective function are necessary for one main step of this optimal control algorithm, composed by the determination of the direction of descent d plus the line search (determination of the step α).

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