Optimal control of the cylinder wake flow using Proper Orthogonal Decomposition (POD)

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Outline

I - Configuration and numerical method

II - Proper Orthogonal Decomposition (POD)

III - Reduced Order Model of the cylinder wake (ROM)

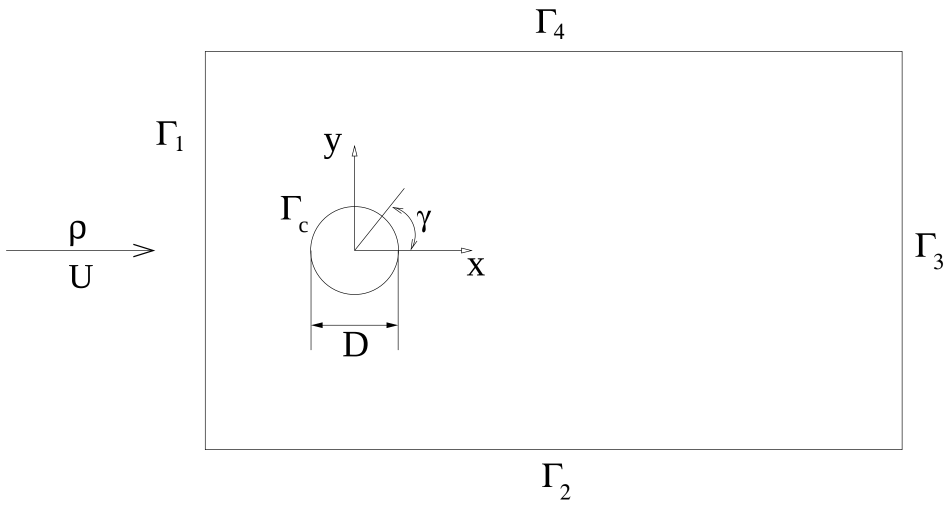
IV - Optimal control formulation based on the reduced order model

V - Closed loop results

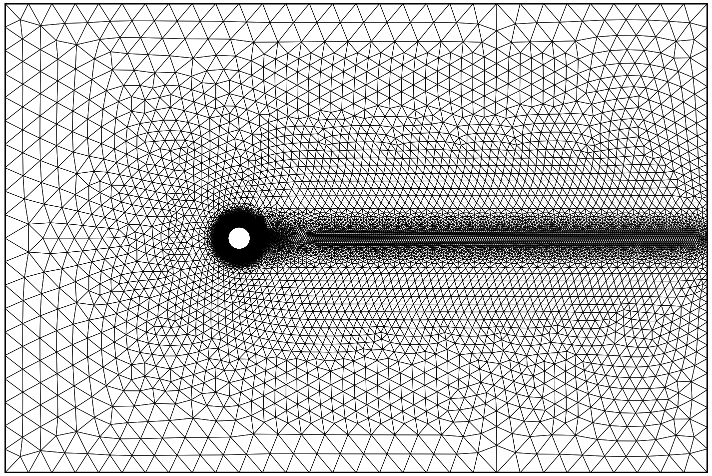
Conclusions and perspectives

I - Configuration and numerical method

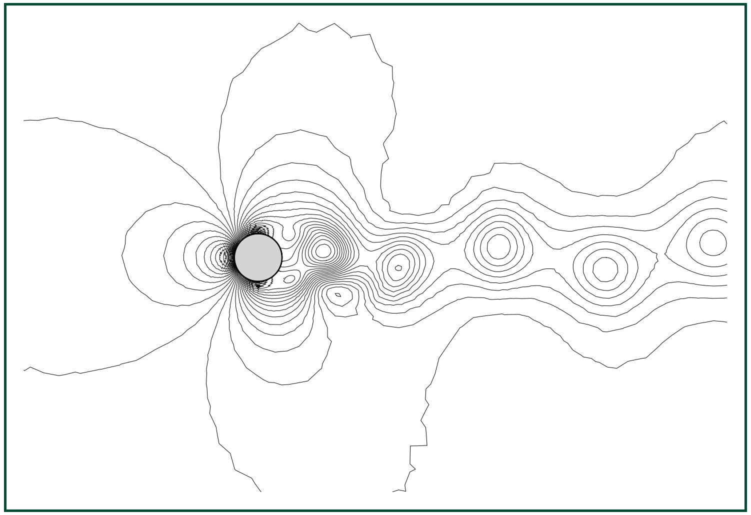
- Two dimensional flow around a circular cylinder at $R_e = 200$
- Viscous, incompressible and Newtonian fluid
- Cylinder oscillation with a tangential velocity $\gamma(t)$



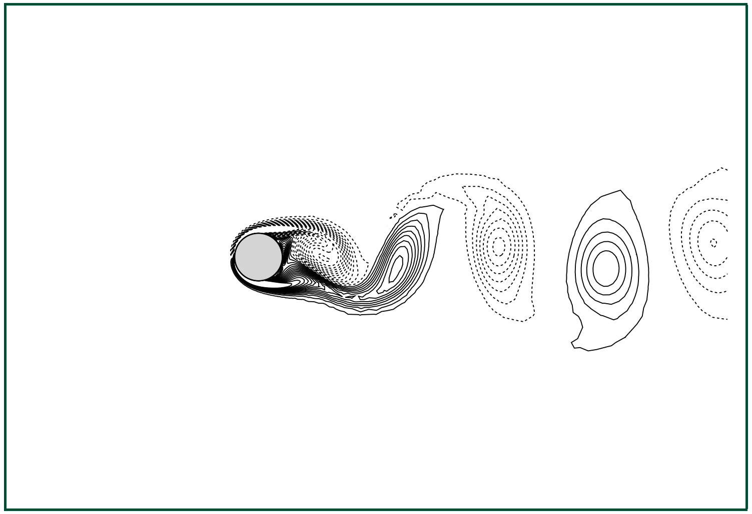
- Fractional steps method in time
- Finite Elements Method (FEM) in space



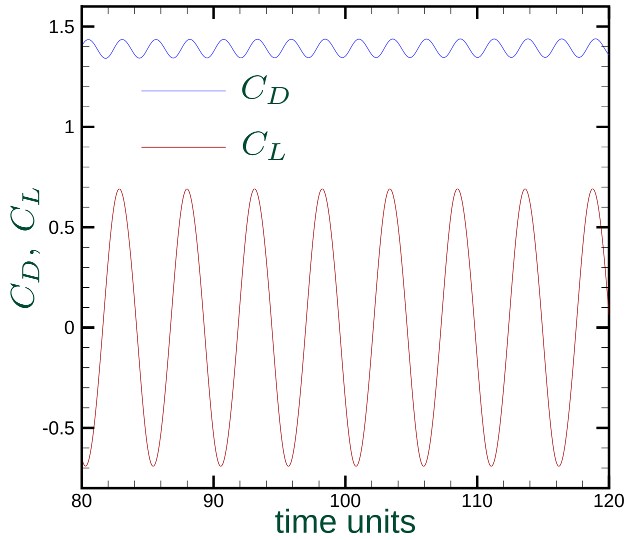
I - Configuration and numerical method



Iso pressure at $t = 100$.



Iso vorticity at $t = 100$.



Aerodynamic coefficients.

Authors	S_t	C_D
Braza *et al.* (1986)	0.2000	1.4000
Henderson *et al.* (1997)	0.1971	1.3412
He *et al.* (2000)	0.1978	1.3560
this study	**0.1983**	**1.3972**

Strouhal number and drag coefficient.

II - Proper Orthogonal Decomposition (POD)

► Introduced in fluid mechanics (turbulence context) by Lumley (1967).

► Look for a realization $\phi(\boldsymbol{X})$ which is closer, in an average sense, to the realizations $\boldsymbol{u}(\boldsymbol{X})$. ($\boldsymbol{X} = (\boldsymbol{x}, t) \in \mathcal{D} = \Omega \times \mathbb{R}^+$)

► $\phi(\boldsymbol{X})$ solution of the problem : $\displaystyle \max_{\boldsymbol{\phi}} \frac{\langle |(\boldsymbol{u}, \boldsymbol{\phi})|^2 \rangle}{\|\boldsymbol{\phi}\|^2}.$

► Snapshots method, Sirovich (1987) :

$$\int_T C(t, t') a^{(n)}(t')\, dt' = \lambda^{(n)} a^{(n)}(t).$$

► Optimal convergence L^2 *norm* (energy) of $\phi(\boldsymbol{X})$

$\Rightarrow$ Dynamical order reduction is possible.

► Decomposition of the velocity field :

$$\boldsymbol{u}(\boldsymbol{x}, t) = \sum_{i=1}^{N_{POD}} a^{(i)}(t) \boldsymbol{\phi}^{(i)}(\boldsymbol{x}).$$

- Galerkin projection of *NSE* on the POD basis :

$$\left(\boldsymbol{\phi}^{(i)},\, \frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u}\cdot\boldsymbol{\nabla})\boldsymbol{u}\right) = \left(\boldsymbol{\phi}^{(i)},\, -\boldsymbol{\nabla} p + \frac{1}{Re}\Delta \boldsymbol{u}\right).$$

- Integration by parts (Green's formula) leads :

$$\left(\boldsymbol{\phi}^{(i)},\, \frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u}\cdot\boldsymbol{\nabla})\boldsymbol{u}\right) = \left(p,\, \boldsymbol{\nabla}\cdot\boldsymbol{\phi}^{(i)}\right) - \frac{1}{Re}\left((\boldsymbol{\nabla}\otimes\boldsymbol{\phi}^{(i)})^T,\, \boldsymbol{\nabla}\otimes\boldsymbol{u}\right) - [p\,\boldsymbol{\phi}^{(i)}] + \frac{1}{Re}[(\boldsymbol{\nabla}\otimes\boldsymbol{u})\boldsymbol{\phi}^{(i)}].$$

with $[\boldsymbol{a}] = \int_\Gamma \boldsymbol{a}\cdot\boldsymbol{n}\, d\Gamma$ and $(A,\, B) = \int_\Omega A : B\, d\Omega = \sum_{i,\,j}\int_\Omega A_{ij}B_{ji}\, d\Omega$.

III - Reduced Order Model of the cylinder wake (ROM)

▶ Velocity decomposition with N_{POD} modes :

$$\boldsymbol{u}(\boldsymbol{x},t) = \boldsymbol{u}_m(\boldsymbol{x}) + \gamma(t)\,\boldsymbol{u}_c(\boldsymbol{x}) + \sum_{k=1}^{N_{POD}} a^{(k)}(t)\boldsymbol{\phi}^{(k)}(\boldsymbol{x}).$$

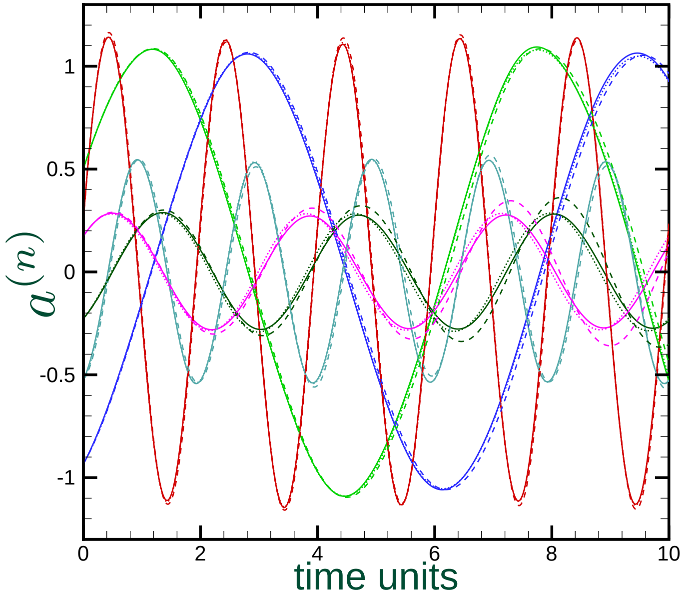
▶ Reduced order dynamical system where only N_{gal} $(\ll N_{POD})$ modes are retained (state equations) :

$$\left\{\begin{array}{l} \dfrac{d\,a^{(i)}(t)}{d\,t} = \mathcal{A}_i + \displaystyle\sum_{j=1}^{N_{gal}} \mathcal{B}_{ij}\,a^{(j)}(t) + \sum_{j=1}^{N_{gal}}\sum_{k=1}^{N_{gal}} \mathcal{C}_{ijk}\,a^{(j)}(t)a^{(k)}(t) \\ \qquad + \mathcal{D}_i\,\dfrac{d\,\gamma}{d\,t} + \left(\mathcal{E}_i + \displaystyle\sum_{j=1}^{N_{gal}} \mathcal{F}_{ij}\,a^{(j)}(t)\right)\gamma + \mathcal{G}_i\gamma^2 \\ a^{(i)}(0) = (\boldsymbol{u}(\boldsymbol{x},\,0),\,\boldsymbol{\phi}^{(i)}(\boldsymbol{x})). \end{array}\right.$$

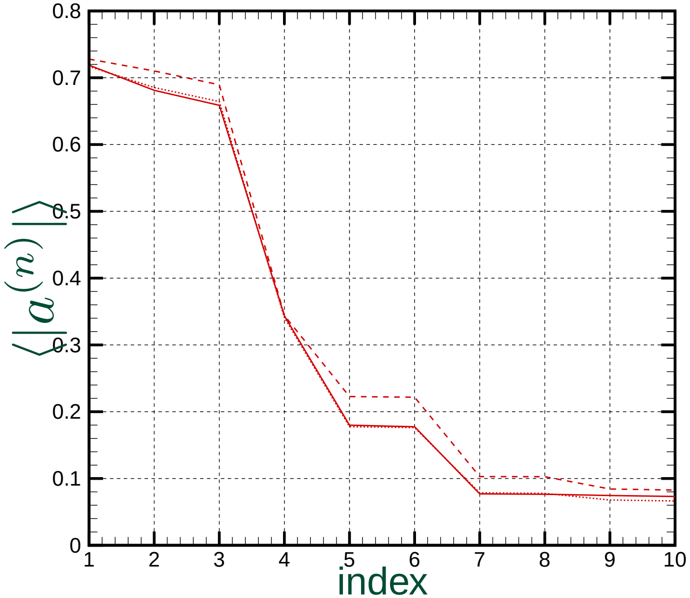
$\mathcal{A}_i$, $\mathcal{B}_{ij}$, $\mathcal{C}_{ijk}$, $\mathcal{D}_i$, $\mathcal{E}_i$, $\mathcal{F}_{ij}$ and $\mathcal{G}_i$ depend on $\boldsymbol{\phi}$, $\boldsymbol{u}_m$, $\boldsymbol{u}_c$ and Re.

III - Reduced Order Model of the cylinder wake (ROM) *Stabilization*

Integration and (optimal) stabilization of the reduced order dynamical system with $\gamma = A\sin(2\pi S_t t)$, $A = 2$ and $S_t = 0,5$.



Temporal evolution of the 6 *first POD modes.*



Average amplitudes of POD modes.

—— projection (DNS)

– – prediction before stabilization (low order model)

· · · prediction after stabilization (low order model).

- Objective functional :

$$\mathcal{J}(\boldsymbol{a},\gamma(t)) = \int_0^T J(\boldsymbol{a},\gamma(t))\,dt = \int_0^T \left(\sum_{i=1}^{N_{gal}} {a^{(i)}}^2 + \frac{\alpha}{2}\gamma(t)^2 \right)\,dt.$$

α : regularization parameter (penalization).

- Adjoint equations :

$$\begin{cases} \dfrac{d\,\xi^{(i)}(t)}{dt} = -\displaystyle\sum_{j=1}^{N_{gal}} \left(\mathcal{B}_{ji} + \gamma\,\mathcal{F}_{ji} + \sum_{k=1}^{N_{gal}} (\mathcal{C}_{jik} + \mathcal{C}_{jki})\,a^{(k)} \right) \xi^{(j)}(t) - 2a^{(i)} \\ \xi^{(i)}(T) = 0. \end{cases}$$

- Optimality condition (gradient) :

$$\delta\gamma(t) = -\sum_{i=1}^{N_{gal}} \mathcal{D}_i \frac{d\xi^{(i)}}{dt} + \sum_{i=1}^{N_{gal}} \left(\mathcal{E}_i + \sum_{j=1}^{N_{gal}} \mathcal{F}_{ij} a^{(j)} + 2\mathcal{G}_i \gamma(t) \right) \xi^{(i)} + \alpha\gamma.$$

IV - Optimal control formulation based on reduced order model

▶ $\gamma^{(0)}(t)$ done ; for $n = 0, 1, 2,$ ... and while a convergence criterium is not satisfied, do :

1. From $t = 0$ to $t = T$ solve the state equations with $\gamma^{(n)}(t)$;
   $\hookrightarrow$ *state variables* $a^{(n)}(t)$

2. From $t = T$ to $t = 0$ solve the adjoint equations with $a^{(n)}(t)$;
   $\hookrightarrow$ *adjoint variables* $\xi^{(n)}(t)$

3. Solve the optimality condition with $a^{(n)}(t)$ and $\xi^{(n)}(t)$;
   $\hookrightarrow$ *objective gradient* $\delta\gamma^{(n)}(t)$

4. New control law $\hookrightarrow \gamma^{(n+1)}(t) = \gamma^{(n)}(t) + \omega^{(n)}\,\delta\gamma^{(n)}(t)$

▶ End do.

V - Closed loop results *Generalities*

▶ No reactualization of the POD basis.

▶ The energetic representativity is *a priori* different to the dynamical one :

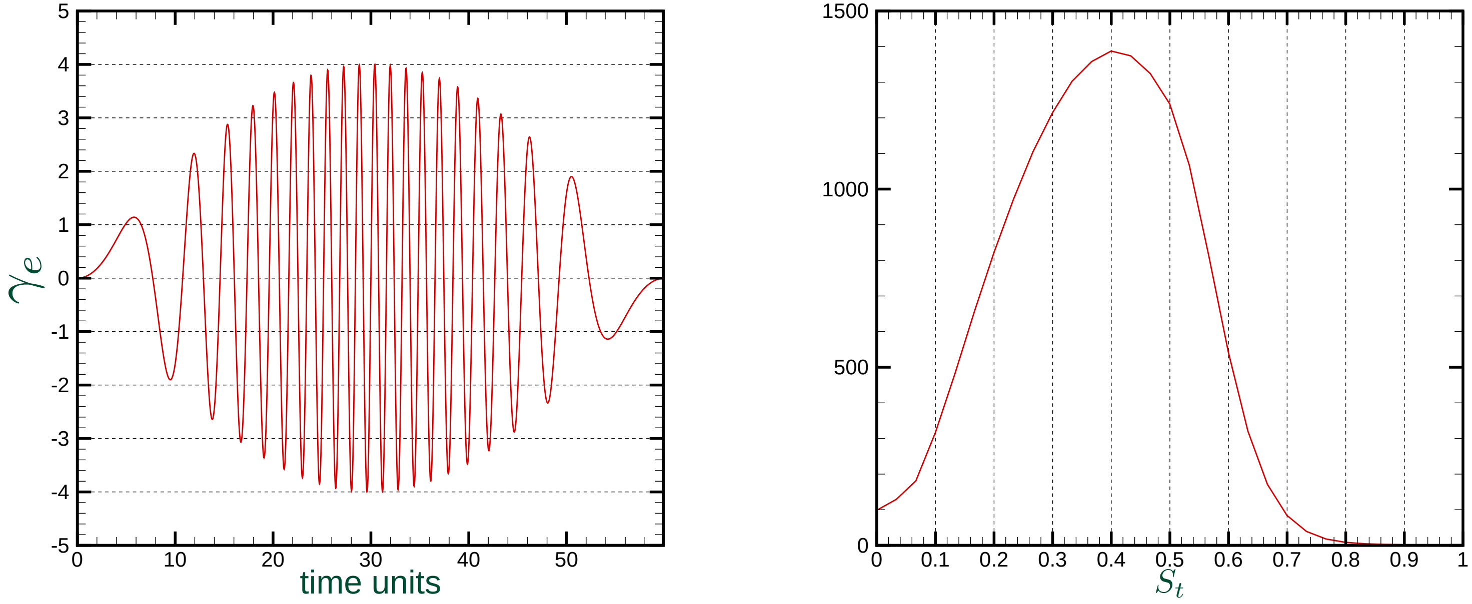
$\hookrightarrow$ possible inconvenient for control,

$\hookrightarrow$ a POD dynamical system represents *a priori* only the dynamics (and its vicinity) used to build the low dynamical model.

▶ Construction of a POD basis representative of a large range of dynamics :

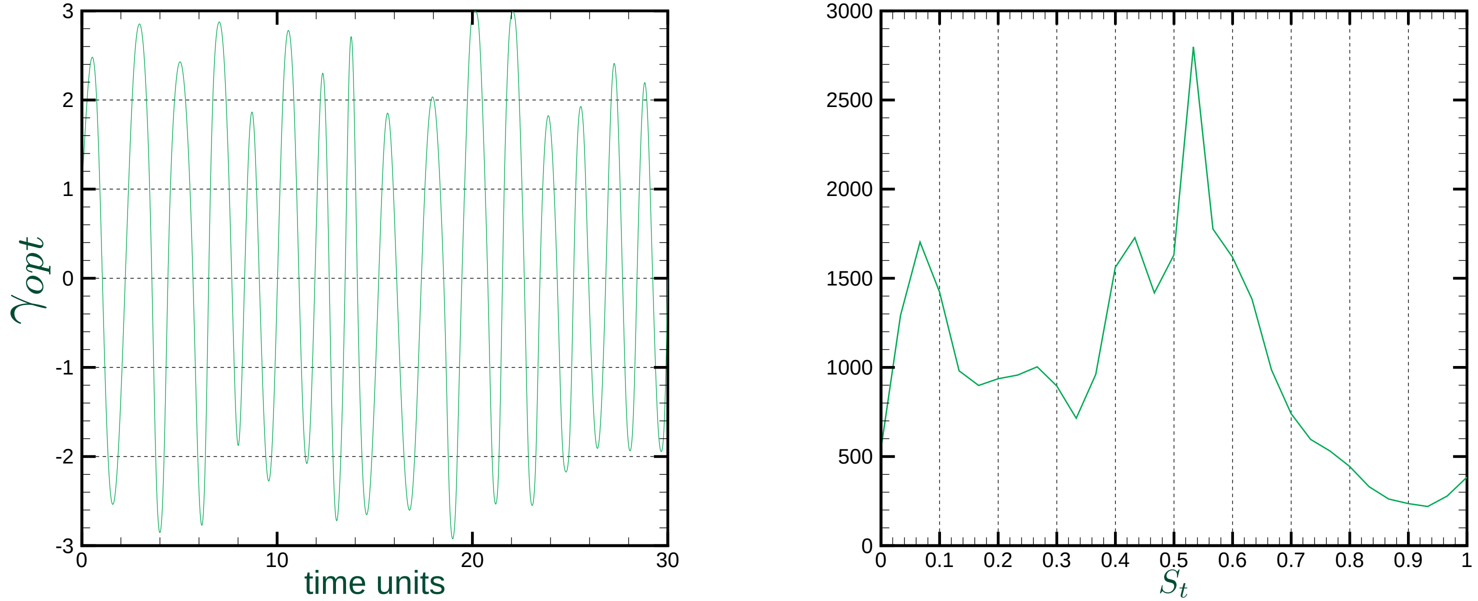
$\hookrightarrow$*excitation of a great number of degrees of freedom scanning* $\gamma(t)$ *in amplitudes and frequencies.*

V - Closed loop results *Excitation*



- $\gamma = 0$:
  $\hookrightarrow$ 2 *modes out of 100 are sufficient to represent* 97% *of the kinetic energy.*

- $\gamma = \gamma_e$:
  $\hookrightarrow$ 30 *modes out of 100 are then necessary to represent* 97% *of the kinetic energy.*



► Reduction of the wake instationarity. $\gamma_{opt} \simeq A\sin(2\pi S_t t)$ with $A = 2.2$ and $S_t = 0.53$

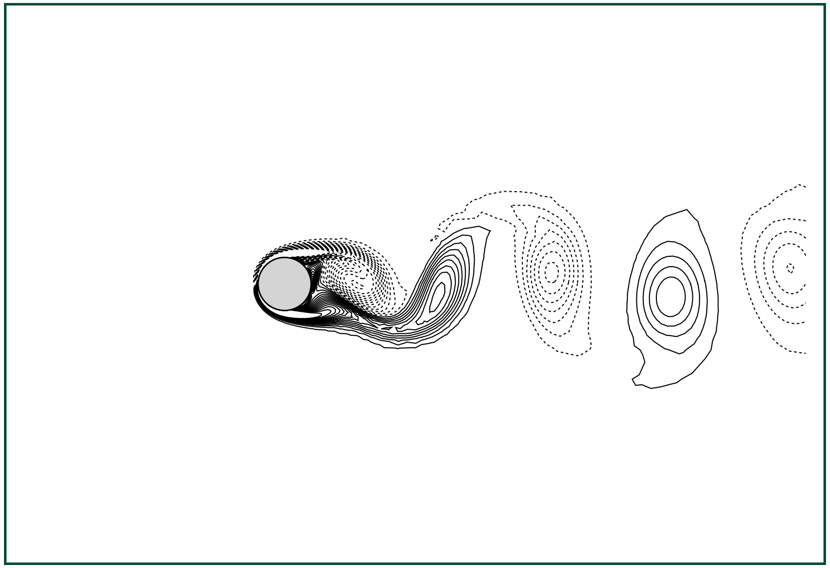
$$\mathcal{J}(\gamma_e) = 9.81 \quad \Longrightarrow \quad \mathcal{J}(\gamma_{opt}) = 5.63.$$

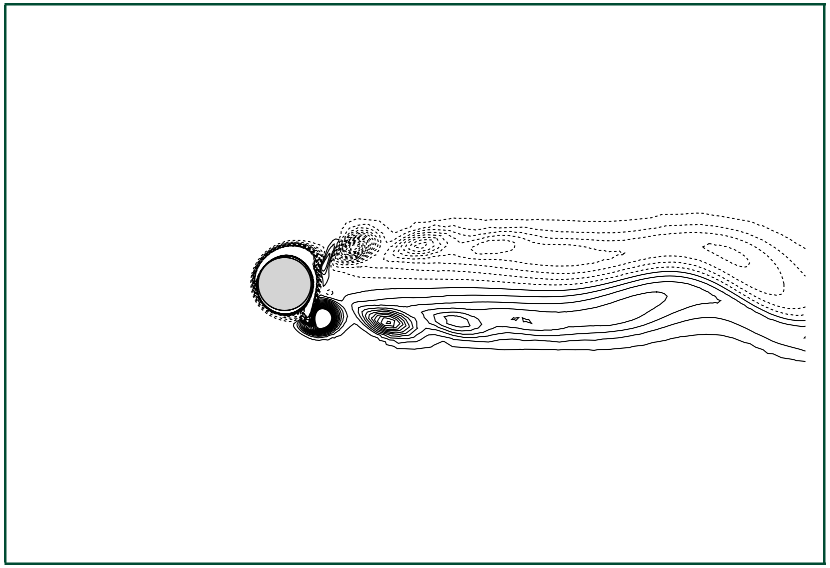
► The control is optimal for the reduced order model based on POD.

► Is it also optimal for the Navier Stokes model ?

V - Closed loop results *Comparison of wakes' organization*

► No mathematical proof concerning the Navier Stokes optimality.





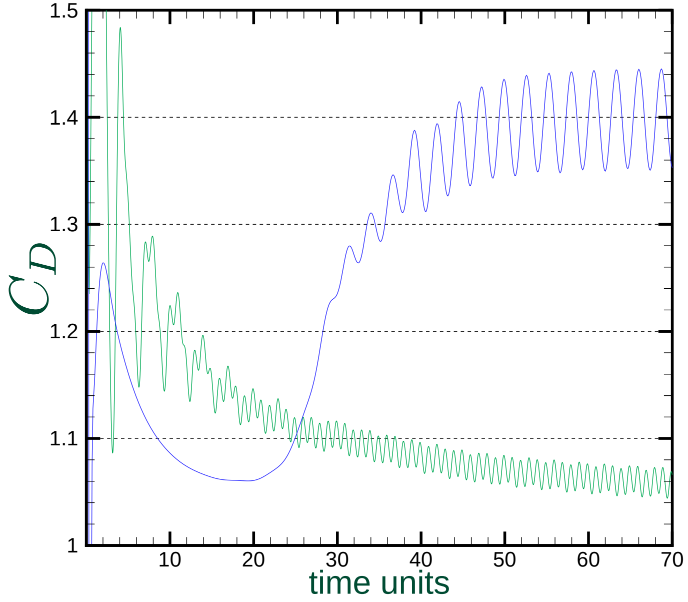
a) no control $\gamma = 0$ *b) optimal control* $\gamma = \gamma_{opt}$

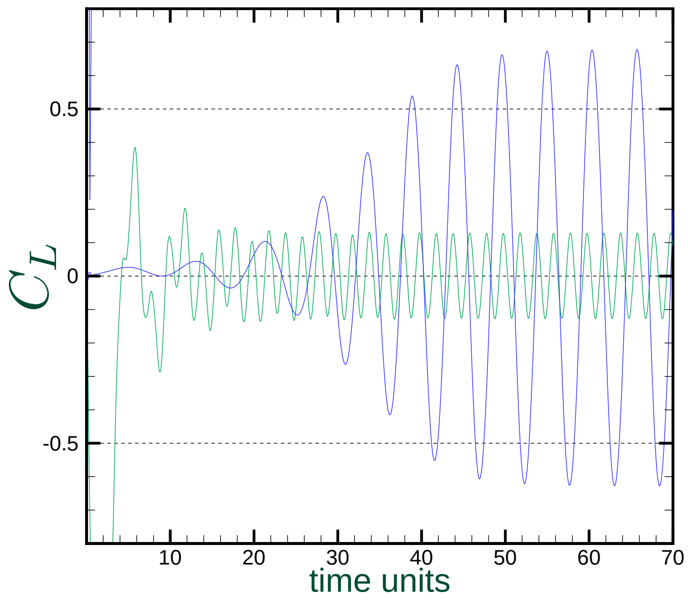
Isocontours of vorticity ω_z.

► no control : $\gamma = 0 \Rightarrow$ Asymmetrical flow.
$\hookrightarrow$ Large and energetic eddies.

► optimal control : $\gamma = \gamma_{opt} \Rightarrow$ Symmetrization of the (near) wake.
$\hookrightarrow$ Smaller and lower energetic eddies.

V - Closed loop results *Aerodynamic coefficients*





- Very consequent drag reduction :
  $C_D = 1.40$ for $\gamma = 0$ et $C_D = 1.06$ for $\gamma = \gamma_{opt}$ (more than 25%).
- Decrease of the lift amplitude :
  $C_L = 0.68$ for $\gamma = 0$ et $C_L = 0.13$ for $\gamma = \gamma_{opt}$.

Conclusions and perspectives

▶ Conclusions

- Significative drag reduction minimizing the wake instationnarity of the ROM.
- Numerical costs (CPU and memory) negligible.

▶ Perspectives

- Improve the representativity of the low order model.
  $\hookrightarrow$ "Optimize" the temporal excitation γ_e,
  $\hookrightarrow$ Mix snapshots corresponding to several differents dynamics (temporal excitations).
- Look for harmonic control $\gamma(t) = A \sin(2\pi\, S_t\, t)$ with POD basis reactualization.
- Couple this optimal system with trust region methods (TRPOD) $\Longrightarrow$ proof of convergence.
- Couple pressure with the POD dynamical system.
- Optimal control of the Navier Stokes equations.