Self-propelled motions of solids in a fluid: mathematical analysis, simulation and control

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The aim of this presentation is to highlight some recent advances on the mathematical analysis and the control of self-propelled motions of solids in a fluid. We study a model consisting in a solid undergoing an undulatory deformation, which is immersed in a viscous incompressible fluid. The motion of the fluid is governed by the incompressible Navier-Stokes equations and the standard conservation's laws of linear and angular momentum rule the dynamics of the structure. The time variation of the fluid domain (due to the motion of the structure) is not known a priori, so we deal with a free boundary value problem. The displacement of the solid is decomposed into a rigid part and a deformation (undulatory) part. The rigid part of the displacement results from the interaction of the fluid and the solid, whereas the deformation part is given. Since our aim is to possibly consider several immersed solids, the domain filled by the fluid is one of the unknowns. Therefore we have to tackle a *free boundary value problem*. The solutions are controlled by an input which is the shape of the solid.

We first show that the initial and boundary value problem obtained by coupling the Navier-Stokes equations for the fluid to Newton's law for the creature is well-posed in Sobolev type spaces. We next give an approximation scheme for the governing equations which is tested on some undulatory motions observed by the zoologists in order to get straight-line-swimming or turning. This scheme is based on a fixed mesh in space and on teh characteristics method in time. We finally tackle, from a control theoretic perspective the swimming of aquatic microorganisms. Since, the Reynolds number is this time very low, we consider a model based on the Stokes equations for the fluid.

This presentation is essentially based on results from [1], [2] and [3].

References

[1] J. San Martín, J.-F. Scheid, T. Takahashi, and M. Tucsnak, *Convergence of the Lagrange-Galerkin method for the equations modelling the motion of a fluid-rigid system*, SIAM J. Numer. Anal., 43 (2005), pp. 1536–1571.

[2] J. A. San Martín, J.-F. Scheid, T. Takahashi, M. Tucsnak, *An Initial and Boundary Value Problem Modeling Fish-like Swimming*, Archive for Rational Mechanics and Analysis, **188** (2008), 429–455.

[3] J. S. Martin, T. Takahashi, M. Tucsnak, *A control theoretic approach to the swimming of microscopic organisms*, Quarterly of Applied Mathematics, **65** (2007), 405–424.