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GTEM Kick-Off Seminar October 2006

p-Groups and Automorphism groups of curves in char. p > 0

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Références

► *R* is a strictly henselian DVR of inequal characteristic (0, *p*).

 $K:=\operatorname{Fr} R$; for example K/\mathbb{Q}_p^{ur} fi nite.

 π a uniformizing parameter.

 $k := R_K/\pi R_K$.

- ▶ C has potentially good reduction over K if there is L/K (fi nite) such that $C \times_K L$ has a smooth model over R_L . Then :
- ► There is a minimal extension *L/K* with this property; it is Galois and called the **monodromy** extension.
- ▶ Gal(L/K) is the monodromy group.
- Its p-Sylow subgroup is the wild monodromy group.

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C/K smooth projective curve, $g(C) \ge 1$.

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- ▶ There is a minimal extension L/K with this property; it is Galois and called the monodromy extension.
- ▶ Gal(L/K) is the monodromy group.
- Its p-Sylow subgroup is the wild monodromy group.

- Let ℓ be a prime number, then, $n_{\ell} := v_{\ell}(|\operatorname{Gal}(L/K)|) \leq v_{\ell}(|\operatorname{Aut}_k C_s|)$
- ▶ If $\ell \notin \{2, p\}$, then $n_{\ell} \leq 2g$.
- If p > 2, then $n_p \le \inf_{\ell \ne 2, p} v_p(|\mathrm{GL}_{2g}(\mathbb{Z}/\ell\mathbb{Z})|) = a + [a/p] + ...$, where $a = [\frac{2g}{p-1}]$.
- This gives an exponential type bound in g for |Aut_kC_s|. This justifi es our interest in looking a polynomial bounds, Stichtenoth ([St 73]) and Singh ([Si 73]).

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- ▶ $G_{\infty,1}(f) := \{ \sigma \in \operatorname{Aut}_k C_f \mid v_{\infty}(\sigma(z) z) \ge 2 \}$, the p-Sylow.
- ▶ ([St 73]) Let $g(C_f) \ge 2$, then $G_{\infty,1}(f)$ is a p-Sylow of $\operatorname{Aut}_k C_f$.
- ▶ It is normal except for $f(X) = X^m$ where m|1 + p.

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- Let $\rho(X) = X$, $\rho(W) = W + 1$, then $< \rho >= G_{\infty,2} \subset Z(G_{\infty,1})$
- ▶ 0 \rightarrow < ρ > \rightarrow $G_{\infty,1}$ \rightarrow V \rightarrow 0, $V = \{\tau_y | \tau_y(X) = X + y, \ y \in k\}.$ $f(X + y) = f(X) + f(y) + (F - \operatorname{Id})(P(X, y)),$ $P(X, y) \in Xk[X].$
- ▶ Let $\tau_y(W) := W + a_y + P(X, y)$, $a_y \in \mathbb{F}_p$, ther $[\tau_y, \tau_z] = \rho^{\epsilon(Y, z)}$, where $\epsilon : V \times V \to \mathbb{F}_p$ is an alternating form.
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- ϵ is non degenerated iff $< \rho >= Z(G_{\infty,1})$.

Lemma

If $f(X) = \sum_{1 \le i \le m} t_i X^i \in k[X]$ is monic, then :

▶ $\Delta(f)(X, Y) := f(X + Y) - f(X) - f(Y) = R(X, Y) + (F - \operatorname{Id})(P_f(X, Y)),$ where $R \in \bigoplus_{\lfloor \frac{m}{p} \rfloor \leq ip^{n(i)} < m, \ (i,p)=1} k[Y]X^{ip^{n(i)}}$ and $P_f \in Xk[X, Y].$

- $ightharpoonup P_f = (\mathrm{Id} + F + ... + F^{n-1})(\Delta(f)) \mod X^{[\frac{m-1}{\rho}]+1}.$
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Let $m-1=\ell p^s$ with $(\ell,p)=1$.

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Characterization of $G_{\infty,1}(f)$

- ▶ We consider the extensions of type $0 \to N \simeq Z/p\mathbb{Z} \to G \to (\mathbb{Z}/p\mathbb{Z})^n \to 0$ (note that $G_{\infty,1}(f)$ is an extension of this type). Then $G' \subset N \subset Z(G)$.
- ▶ If G' = Z(G), G is called extraspecial.

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 - If p > 2, we denote by $E(p^3)$ (resp. $M(p^3)$) the non abelian group of order p^3 and exponent p (resp. p^2). Then, $G \simeq E(p^3) * E(p^3) * ... * E(p^3)$ or $M(p^3) * E(p^3) * ... * E(p^3)$, according as the exponent is p or p^2 .
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([Le-Ma 1]). Let
$$f(X) = X\Sigma(F)(X) \in Xk[X]$$
, $\Sigma(F) = \sum_{0 \le i \le s} a_i F^i \in k\{F\}$ an additive polynomial with deg $f = 1 + p^s$. Then,

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▶ Theorem

([Le-Ma 1]). Let $f(X) \in Xk[X]$ with $(\deg f, p) = 1$. If $\frac{|G_{\infty,1}|}{q}>rac{p}{p-1}$ ($rac{2}{3}$ for p=2), then $f(X) = cX + X\Sigma(F)(X) \in k[X].$

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▶ Defi nition

Let (C, G) with $G \subset \operatorname{Aut}_k C$. We say that (C, G) is a **big** action if G is a p-group and |G| > 2p

(N) $g_C > 0$ and $\frac{|G|}{g_C} > \frac{2p}{p-1}$.

It follows from ([Na 87]) that there is $\infty \in C$, with

- $C \to C/G \simeq \mathbb{P}^1_k \infty$ is étale and $G = G_{\infty,1}$.
- $lackbox{G}_{\infty,2}
 eq G_{\infty,1}$ and $C/G_{\infty,2}\simeq \mathbb{P}_k^1$
- ► Then, $G_{\infty,1}/G_{\infty,2}$ acts as a group of translations
 - of the affi ne line $C/G_{\infty,2} \{\infty\}$
- ▶ Transfert of condition (N) to quotients. Let (C, G) a big action, if $H \triangleleft G$ and if g(C/H) > 0

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- ▶ If $G_2 \simeq (\mathbb{Z}/p\mathbb{Z})^t$, then $k(C) = k(X, W_1, ..., W_t)$ and $\wp(W_1, ..., W_t) = (f_1(X), f_2(X), ..., f_t(X)) \in (k[X])^t$ $f_1(X), ..., f_t(X)$ are \mathbb{F}_p -free mod $\wp(k[X])$.
- ► The group extension
 - $0 \to G_2 \to G_1 \to V = (\mathbb{Z}/p\mathbb{Z})^V \to 0$ induces a representation $\rho: V \to \mathrm{Gl}_t(\mathbb{F}_p)$

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- The group extension 0 → G₂ → G₁ → V = (ℤ/pℤ)^v → 0 induces a representation ρ : V → Gl_t(𝔽_p)

▶ For p > 2, we give an example such that $\text{Im} \rho \neq \text{Id}$

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Páfárancas

▶ For p > 2, we give an example such that $\text{Im} \rho \neq \text{Id}$

Let
$$f_1 := X(\alpha F)(X) = \alpha X^{1+p}$$
 with $\alpha^p + \alpha =$
then $\mathrm{Ad}_{f_1} = Y^{p^2} - Y$ and
Let $f_2 := X^{1+2p} - X^{2+p}$, then

▶ If $y \in Z(Ad_{f_1}) = \mathbb{F}_{n^2}$ one has

$$f_2(X+y) = \frac{2(y^p-y)}{\alpha}f_1(X) + f_2(X) + \wp(P_2)$$

where $y \to \frac{2(y^p - y)}{\alpha}$ is a non zero linear form over

$$|G| = p^2 p^2 \text{ and } g = \frac{p-1}{2} (p+p*2p)$$

$$\frac{|G|}{g} = \frac{2p}{p-1} \frac{p^2}{1+2p}.$$

 $\frac{|G|}{g^2} = \frac{4p}{(p-1)^2} \frac{p}{(1+2p)^2}$

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- ▶ For p > 2, we give an example such that $\text{Im} \rho \neq \text{Id}$
 - Let $f_1 := X(\alpha F)(X) = \alpha X^{1+p}$ with $\alpha^p + \alpha = 0$; then $\mathrm{Ad}_{f_1} = Y^{p^2} - Y$ and Let $f_2 := X^{1+2p} - X^{2+p}$, then
 - If $y \in Z(\mathrm{Ad}_{f_1}) = \mathbb{F}_{p^2}$ one has $f_2(X+y) = \frac{2(y^p-y)}{\alpha}f_1(X) + f_2(X) + \wp(P_2).$ where $y \to \frac{2(y^p-y)}{\alpha}$ is a non zero linear form ove \mathbb{F}_{p^2} with values in \mathbb{F}_{p^2} .

|
$$G| = p^2 p^2$$
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$$|G| = p^2 p^2$$
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 - $|G| = p^2 p^2 \text{ and } g = \frac{p-1}{2} (p+p*2p).$ $\frac{|G|}{g} = \frac{2p}{p-1} \frac{p^2}{1+2p}.$ $\frac{|G|}{g^2} = \frac{4p}{(p-1)^2} \frac{p}{(1+2p)^2}.$

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▶ Theorem

([Le-Ma 4]) Assume G_2 is non abelian, then $G_2=G^\prime$.

- ▶ Sketch proof : If $G' \neq G_2$, there is $H \triangleleft G$ with $G' \subseteq H \subseteq G_2$ and $[G_2 : H] = p$. (C/H, G/H) is a big action ;
- ► $C/H : W^p W = f := X\Sigma(F)(X),$ $\deg(f) = 1 + p^s.$
- ▶ $(AutC/H)_{\infty,1} := E$ is extraspecial of order p^{2s+1} .
- ► G/H is abelian and normal in E
- ► ([Hu 67] Satz 13.7 p. 353) $|G/H| \le p^{s+1}$ and so $|G/H|/g(C/H) \le \frac{2p^{s+1}}{(p-1)p^s} = \frac{2p}{p-1}$, a contradiction
- We deduce the following corollary from ([Su 86 4.21 p.75).

Corollary

If $|G_2| = p^3$, then G_2 is abelian.

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- ▶ ([Hu 67] Satz 13.7 p. 353) $|G/H| \le p^{s+1}$ and so $|G/H|/g(C/H) \le \frac{2p^{s+1}}{(p-1)p^s} = \frac{2p}{p-1}$, a contradiction
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- $(\operatorname{Aut} C/H)_{\infty,1} := E$ is extraspecial of order p^{2s+1} .
- ► G/H is abelian and normal in E
- ([Hu 67] Satz 13.7 p. 353) $|G/H| \le p^{s+1}$ and so $|G/H|/g(C/H) \le \frac{2p^{s+1}}{(p-1)p^s} = \frac{2p}{p-1}$, a contradiction
- ▶ We deduce the following corollary from ([Su 86 4.21 p.75).

Corollary

If $|G_2| = p^3$, then G_2 is abelian

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- ▶ Sketch proof : If $G' \neq G_2$, there is $H \triangleleft G$ with $G' \subset H \subset G_2$ and $[G_2 : H] = p$. (C/H, G/H) is a big action ;
- $C/H: W^p W = f := X\Sigma(F)(X),$ $\deg(f) = 1 + \rho^s.$
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([Le-Ma 4]) Assume G_2 is non abelian, then $G_2=G^\prime$.

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- ▶ In characteristic 0, an analogue of big actions is given by the actions of a fi nite group G on a compact Riemann surface C with $g_C \ge 2$ such that $|G| = 84(g_C 1)$ (we say that C is an **Hurwitz curve**) ([Co 90]).
- Let us mention Klein's quartic ($G \simeq PSL_2(\mathbb{F}_7)$) ([El 99]).
- ▶ The Fricke-Macbeath curve with genus 7 $(G \simeq PSL_2(\mathbb{F}_8))$ ([Mc] 65).
- Let C be an Hurwitz curve with genus g_C . Let n > 1 and C_n the maximal unramified Galois cover whose group is abelian with exponent n. The Galois group of C_n/C is $(\mathbb{Z}/n\mathbb{Z})^{2g_C}$. It follows from the unicity of C_n that the k-automorphisms of C have n^{2g} prolongations to C_n . It follows that C_n is an Hurwitz curve $(M_C \setminus G_n)$.

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If (C, G) is a big action then C → C/G is an étale cover of the affi ne line whose group is a p-group; it follows that the Hasse-Witt invariant of C is zero; therefore, in order to adapt the previous proof to char. p > 0, one needs to accept ramifi cation. This is done with the so called ray class fields of function fields over finite fields.

▶ Let $K := \mathbb{F}_q(X)$ where $q = p^e$, S the set of fi nite rational places (X - v), $v \in \mathbb{F}_q$ and $m \in \mathbb{N}$. Let K^{alg} be an algebraic closure. Let $K^m_S \subset K^{alg}$, the biggest abelian extension L of K with conductor $\leq m\infty$ and such that the places in S are completely decomposed.

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- ▶ ([Le-Ma 4]) Let C_m/\mathbb{F}_q be the smooth projective curve with function fi eld K_S^m . The translations $X \to X + v$, $v \in \mathbb{F}_q$ stabilize S and ∞ ; they can be extended to \mathbb{F}_q -automorphisms of K_S^m . In this way, we get an action of a p-group G(m) on C_m with $0 \to G_S(m) \to G(m) \to \mathbb{F}_q \to 0$
- ▶ ([Au 00]) If $n_m := |G_S(m)|$, then $g_{C_m} = 1 + n_m(-1 + m/2) (1/2) \sum_{0 \le j \le m-1} n_j \le n_m(-1 + m/2)$

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- Let $N_q := |C_m(\mathbb{F}_q)|$. Then, $N_q = 1 + |G(m)|$, and the quotient $\frac{|G(m)|}{g_{C_m}} \sim \frac{N_q}{g_{C_m}}$.
- ▶ ([La 99]) If $q = p^e$, $m_2 := p^{\lceil e/2 \rceil + 1} + p + 1$ is the smallest conductor m such that the exponent of G_S^m is > p.
- ▶ If e > 2, $(C_{m_2}, G(m_2))$ is a big action and G_2 is abelian with exponent p^2 .

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- Theorem ([Le-Ma 1]) If $\frac{|G|}{g_C^2} \geq \frac{4}{(p-1)^2}$ there is $\Sigma(F) \in k\{F\}$ and $f = cX + X\Sigma(F)(X) \in k[X]$ with $C \simeq C_f$. Moreover there are two possibilities for G:

 $v = \frac{1}{2} = \frac{1}{(p-1)^p}$ and $G \subset G_{\infty,1}(t)$ has index p.

▶ One can push the "classification" of big actions up to the condition $\frac{|G|}{|G|} \ge \frac{4}{10^2 - 41^2}$.

theorem ([Le-Ma 4]) For all M>0, the set $\frac{G}{g_0^2}>M$, for (C,G) a big action with G_2 abelian with exponent p_0 is finite.

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- Let i_0 with $G_2 = G_3 = = G_{i_0} \supseteq G_{i_0+1}$. Then $g_{(C/G_{i_0+1})} = \frac{1}{2}(|G_2/G_{i_0+1}| 1)(i_0 1)$.
- ► Theorem

([Le-Ma 1]) If $\frac{|G|}{g_C^2} \ge \frac{4}{(p-1)^2}$ there is $\Sigma(F) \in k\{F\}$ and $f = cX + X\Sigma(F)(X) \in k[X]$ with $C \simeq C_f$. Moreover there are two possibilities for G:

•
$$\frac{|G|}{g_C^2}=\frac{4p}{(p-1)^2}$$
 and $G=G_{\infty,1}(f)$ or
• $\frac{|G|}{g_2^2}=\frac{4}{(p-1)^2}$ and $G\subset G_{\infty,1}(f)$ has index p .

▶ One can push the "classifi cation" of big actions up to the condition $\frac{|G|}{g_c^2} \ge \frac{4}{(p^2-1)^2}$.

▶ Theorem ([Le-Ma 4]) For all M > 0, the set $\frac{|G|}{g_C^2} > M$, for (C, G) a big action with G_2 abelian with exponent p, is finite.

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- ▶ $f'(Y)/f(Y) = S_1(Y)/S_0(Y), (S_0(Y), S_1(Y)) = 1;$ then $\deg(S_1(Y)) = m - 1$ and $\deg(S_0(Y)) = m$.
- ▶ $f(X + Y) = f(Y)((1 + a_1(Y)X + ... + a_r(Y)X^r)^p \sum_{r+1 \le i \le n} A_i(Y)X^i)$, où r + 1 = [n/p], $a_i(Y), A_i(Y) \in K(Y)$.
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$$v(x_i, p) = 1$$
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Special fi ber of the easy model

We mean the R-model C_R defined by $Z_0^p = f(X_0) = \prod_{1 < i < m} (X_0 - x_i)^{n_i} \in R[X_0]$ (cf. fig 1).

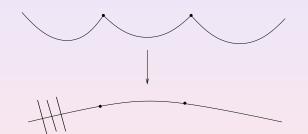


FIG.: $C_R \otimes_R k \longrightarrow \mathbb{P}^1_k$ with singularities and branch locus

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Theorem

([Le-Ma 3])

- ▶ p > 2, $q = p^n$, $n \ge 1$, $K = \mathbb{Q}_p^{\mathrm{ur}}(p^{p/(q+1)})$ and $C \longrightarrow \mathbb{P}_K^1$ is birationally defined by the equation $Z_0^p = f(X_0) = 1 + p^{p/(q+1)}X_0^q + X_0^{q+1}$.
- ► Then, C has potentially good reduction and L(Y) is irreducible over K.
- ▶ The monodromy L/K is the extension of the decomposition field of $\mathcal{L}(Y)$ obtained by adjoining the p-roots $f(y)^{1/p}$, for y describing the zeroes of $\mathcal{L}(Y)$.
- ► The monodromy group is the extraspecial group with exponent p² and order pq² (which is maximal for this conductor).

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Curves of genus 2 ([Le-Ma 3])

- ▶ Case p = 2 and m = 5 (i.e. curves with genus 2 over a 2-adic fi eld $\subset \mathbb{Q}^{\text{tame}}$).
- ► There are 3 types of degeneration for the marked

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Type 1 Type 2
$$\operatorname{Gal}(K'/K)_w \hookrightarrow \operatorname{Q}_8 \times \operatorname{Q}_8 \quad \operatorname{Gal}(K'/K)_w \hookrightarrow (\operatorname{Q}_8 \times \operatorname{Q}_8) \rtimes \mathbb{Z}/2\mathbb{Z}$$

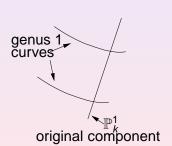
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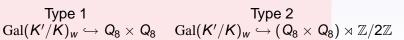
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- ▶ $C \longrightarrow \mathbb{P}_K^1$ is birationally defined by the equation $Z_0^p = f(X_0)$ with $f(X_0) = 1 + b_2X_0^2 + b_3X_0^3 + b_4X_0^4 + X_0^5 \in R[X_0]$.
- Now, we see that the monodromy can be maximal for the 3 types of degeneration.
- a) $f(X_0) = 1 + 2^{3/5}X_0^2 + X_0^3 + 2^{2/5}X_0^4 + X_0^5$ and $K = \mathbb{Q}_2^{\text{ur}}(2^{1/15})$;
- ► C has a marked stable model of type 1
- ▶ The maximal wild monodromy group is $\simeq Q_8 \times Q_8$

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- C has a marked stable model of type 1.
- ▶ The maximal wild monodromy group is $\simeq Q_8 \times Q_8$.

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▶ $C \longrightarrow \mathbb{P}^1_K$ is birationally defined by the equation $Z_0^p = f(X_0)$ with $f(X_0) = 1 + b_2X_0^2 + b_3X_0^3 + b_4X_0^4 + X_0^5 \in R[X_0]$.

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- ▶ b) Let $K = \mathbb{Q}_2^{ur}(a)$ with $a^9 = 2$ and $f(X_0) = 1 + a^{3}X_0^2 + a^{6}X_0^3 + X_0^5$.
- C has a marked stable model of type 2.

• c)
$$K = \mathbb{Q}_2^{\mathrm{ur}}$$
 and $f(X_0) = 1 + X_0^4 + X_0^5$

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- C has a marked stable model of type 2.
- ▶ The maximal wild monodromy group is $\simeq (Q_8 \times Q_8) \rtimes \mathbb{Z}/2\mathbb{Z}$, where $\mathbb{Z}/2\mathbb{Z}$ exchanges the 2 factors
- lacksquare c) $K=\mathbb{Q}_2^{\mathrm{ur}}$ and . $f(X_0)=1+X_0^4+X_0^5$.
- ▶ C has potentially good reduction (i.e. is of type 3
- ► The maximal wild monodromy group is ≃ Q₈ * D₈

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