This document is a list of supplements, errata and clarifications on my publications. Wish it will not have to get too long.[1]

(1) In the article *Représentations cristallines dans le cas d'un corps résiduel imparfait*, Annales de l'Institut Fourier 56, no. 4, pp. 919-999 (2006), it is claimed on p.924 line 5 that the map $i_\sigma\colon \overline{K} \to \overline{\mathbb{K}}$ has dense image, so that it induces an isomorphism $C \simeq \mathbb{C}$. Here is a justification of this fact. Thanks to Hui Gao for a request for clarification and discussion on this point.

Denote by F the Witt vectors Frobenius. Let $m \in \mathbf{N}$: by definition of i_σ, the diagram

$$\begin{array}{ccc} \mathcal{O}_{K_0} & \xrightarrow{\sigma^m} & \mathcal{O}_{K_0} \\ {\scriptstyle i_\sigma}\downarrow & & \downarrow{\scriptstyle i_\sigma} \\ W(\mathbf{k}) & \xrightarrow{F^m} & W(\mathbf{k}) \end{array}$$

is commutative. Denote by $\sigma^{-m}(\mathcal{O}_{K_0})$ the ring $\mathcal{O}_{K_0}$ seen as an $\mathcal{O}_{K_0}$-algebra via σ^m, and $\sigma^{-m}(K_0)$ its fraction field. By the previous commutative diagram, the map

$$F^{-m} \circ i_\sigma\colon \sigma^{-m}(\mathcal{O}_{K_0}) \to W(\mathbf{k})$$

extends $i_\sigma\colon \mathcal{O}_{K_0} \to W(\mathbf{k})$. The family $(\sigma^{-m}(\mathcal{O}_{K_0}))_m$ (with transition maps given by σ) forms an inductive system: let A be its inductive limit (its union). The maps above glue into a map $\tilde{i}_\sigma : A \to W(\mathbf{k})$.

Let $(b_1, \ldots, b_e)$ be a family of elements in $\mathcal{O}_{K_0}$ whose reduction mod p form a p-basis $(\bar{b}_1, \ldots, \bar{b}_e)$ of k_K. The image of $(F^{-m} \circ i_\sigma)(b_i)$ in $\mathbf{k}$ is $\bar{b}_i^{1/p^m}$. This implies that $\tilde{i}_\sigma(A)$ is dense in $W(\mathbf{k})$.

Krasner's lemma thus imply that i_σ extends into a morphism $i_\sigma\colon \overline{K} \to \overline{\mathbb{K}}$ with dense image, which in turn extends into an isomorphism $C \simeq \mathbb{C}$ on completions.

Remark. Note that for each m, the extension $\sigma^{-m}(K_0)/K_0$ is finite of degree p^{me}, so that $A[1/p]$ is algebraic over K_0.

(2) In the article *Filtered (φ, N)-modules and semi-stable representations*, Panoramas et synthèses 54, pp. 93-129 (2019), in Definition 3.23, the condition "for every sub-object $D' \subset D$ in $\mathbf{MF}_K(\varphi, N)$" should be read "for every sub-K_0-vector space $D' \subset D$ stable by φ, endowing D'_K by the filtration induced by that of D_K". Thanks to Léo Poyeton for pointing this inaccuracy.

(3) In the article *Représentations cristallines et F-cristaux : le cas d'un corps résiduel imparfait*, Rendiconti del Seminario Matematico della Università di Padova 119, pp. 141-171 (2008), Proposition 6.8 is wrong (formula (5) in the proof does not hold true). This mistake was discovered by Eike Lau and Tong Liu. This cannot be repaired, as observed and clarified by Hui Gao in *Integral p-adic Hodge theory in the imperfect residue field case* Remark 5.1.4 and Theorem 6.2.3 (`https://arxiv.org/abs/2007.06879`). This implies that the equivalence of Theorem 6.10 is not proved.

(4) In the article *Une généralisation de la théorie de Sen*, Mathematische Annalen 327, pp. 793-813 (2003), it is stated without proof that the groups $\mathrm{Gal}\big(K^{(\infty)}\big[t_1^{(m_1)}, \ldots, t_h^{(m_h)}\big]/K\big[t_1^{(m_1)}, \ldots, t_h^{(m_h)}\big]\big)$ and Γ' are canonically isomorphic. This is not completely obvious, as observed by Hui Gao. Here is a proof.
Let $(\zeta_{p^n})_n \in \overline{K}^{\mathbf{N}}$ be a compatible system of primitive p^n-th roots of unity; put $r = \max\{n \in \mathbf{N}\,;\, \zeta_{p^n} \in K\}$. Let $m \in \mathbf{N}$: the restriction map

$$\mathrm{Gal}\left(K[\zeta_{p^\infty}, t_1^{1/p^m}, \ldots, t_h^{1/p^m}]/K[t_1^{1/p^m}, \ldots, t_h^{1/p^m}]\right) \to \mathrm{Gal}\left(K[\zeta_{p^\infty}]/K\right)$$

is an injective group homomorphism, whose image is $\mathrm{Gal}\left(K[\zeta_{p^\infty}]/F\right)$ where $F = K[\zeta_{p^\infty}] \cap K[t_1^{1/p^m}, \ldots, t_h^{1/p^m}]$. The question is whether the inclusion $K \subset F$ is an equality or not.
Assume it is not. This means that $K[\zeta_{p^{r+1}}] \subset K\big[t_1^{1/p^m}, \ldots, t_h^{1/p^m}\big]$. Reducing m, we may assume that $\zeta_{p^{r+1}} \notin K\big[t_1^{1/p^{m-1}}, \ldots, t_h^{1/p^{m-1}}\big]$. Replacing K by $K\big[t_1^{1/p^{m-1}}, \ldots, t_h^{1/p^{m-1}}\big]$, we thus may assume that $m = 1$.

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If we put $s = \max\{j \in \{1, \dots, h\}\,;\, \zeta_{p^{r+1}} \notin K[t_1^{1/p}, \dots, t_j^{1/p}]\}$: we have $s < h$. Replacing K by $K[t_1^{1/p}, \dots, t_s^{1/p}]$ and putting $t = t_{s+1}$, we may assume that $\zeta_{p^{r+1}} \in K[t^{1/p}] \setminus K$.

As $\big[K[t^{1/p}] : K\big] = p$ and ζ_p has degree $< p$ over K, we have $r > 0$, *i.e.* $\zeta_p \in K$. We have $K[t^{1/p}] = \bigoplus_{i=0}^{p-1} K t^{i/p}$: write $\zeta_{p^{r+1}} = \sum_{i=0}^{p-1} \alpha_i t^{i/p}$ with $\alpha_0, \dots, \alpha_{p-1} \in K$. Let also $\mathrm{Tr} : K[t^{1/p}] \to K$ be the trace map. If $0 < i < p$, the conjugates of $t^{i/p}$ over K are $\zeta_p^j t^{i/p}$ with $j \in \{0, \dots, p-1\}$, so that $\mathrm{Tr}(t^{i/p}) = 0$. Similarly, the conjugates of $\zeta_{p^{r+1}}$ are $\zeta_p^j \zeta_{p^{r+1}}$ with $j \in \{0, \dots, p-1\}$, so $\mathrm{Tr}(\zeta_{p^{r+1}}) = 0$. Taking the trace of the equality $\zeta_{p^{r+1}} = \sum_{i=0}^{p-1} \alpha_i t^{i/p}$ implies that $p\alpha_0 = 0$, ie $\alpha_0 = 0$. More generally, if $\zeta_{p^{r+1}} t^{-i/p} \notin K$, its conjugates over K are obtained by multiplying by powers of ζ_p, so $\mathrm{Tr}(\zeta_{p^{r+1}} t^{-i/p}) = 0$, which in turn implies that $\alpha_i = 0$. This cannot hold for all $i \in \{0, \dots, p-1\}$ (otherwise we would get $\zeta_{p^{r+1}} = 0$): there exists $i \in \{0, \dots, p-1\}$ such that $\zeta_{p^{r+1}} t^{-i/p} \in K$. Note that both $\zeta_{p^{r+1}}$ and $t^{i/p}$ are invertible elements in the ring of integers of $K[t^{1/p}]$: reducing modulo the maximal ideal, we deduce that $\bar{t}^{-i/p}$ whence $\bar{t}^{1/p}$ belong to the residue field of K, which is absurd.

Remark. This fact is not really necessary for Sen's theory: we can reduce to this case where it holds after replacing K by a suitable subextension of $K[\zeta_{p^\infty}, t_1^{1/p^\infty}, \dots, t_h^{1/p^\infty}]$; this does not change the group H_K and replaces Γ_K by a open subgroup, and might only require an extra use of Hilbert 90 at the end.