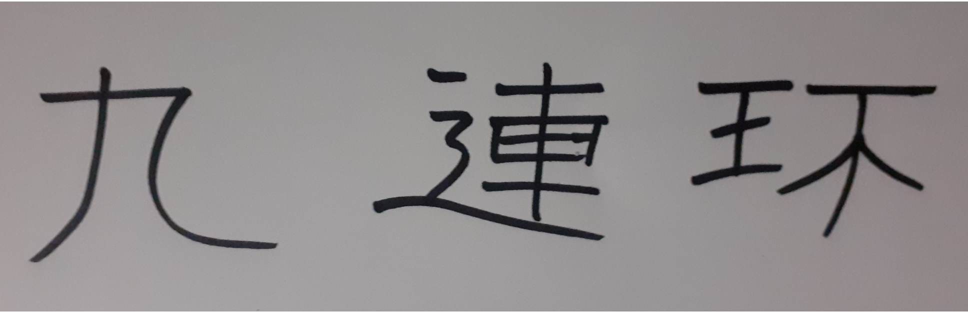
Le baguenaudier

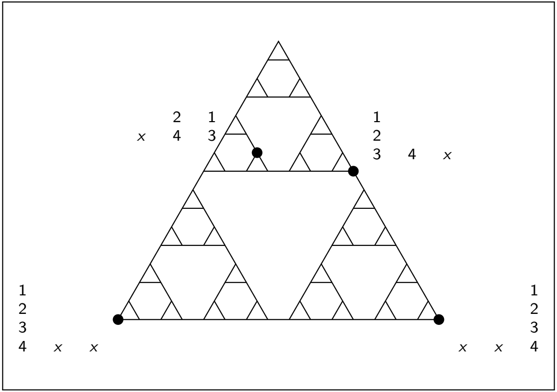
Éric Balandraud



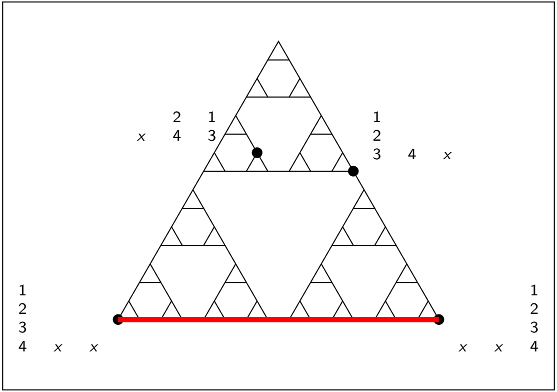
Plan

- En chine médiévale
- Gerolamo Cardano (1501-1576)
  De subtilitate libri XV (1521)
- Édouard Lucas (1842-1891)
  Récréations mathématiques Tome 1 (1892)

Les tours de Hanoï



Les tours de Hanoï



Le code Gray

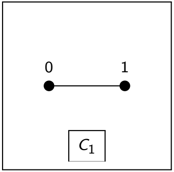
Écrire les entiers sans "phénomène de retenue"
On commence par 000 !!

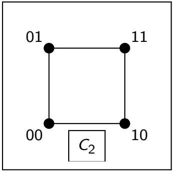
Le code Gray

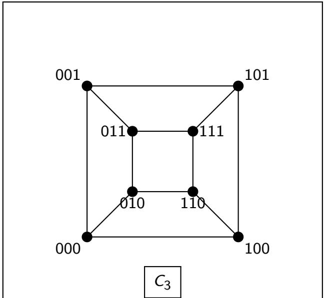
n	Code Gray	Binaire	$A(n)$
0	$\boxed{0000}$	0000	$\emptyset$
1	$000\underline{1}$	0001	$\{0\}$
2	$00\underline{1}1$	0010	$\{0,1\}$
3	$001\underline{0}$	0011	$\{1\}$
4	$0\underline{1}10$	0100	$\{1,2\}$
5	$011\underline{1}$	0101	$\{0,1,2\}$
6	$01\underline{0}1$	0110	$\{0,2\}$
7	$010\underline{0}$	0111	$\{2\}$
8	$\underline{1}100$	1000	$\{2,3\}$
9	$110\underline{1}$	1001	$\{0,2,3\}$
10	$11\underline{1}1$	1010	$\{0,1,2,3\}$
11	$111\underline{0}$	1011	$\{1,2,3\}$
12	$1\underline{0}10$	1100	$\{1,3\}$
13	$101\underline{1}$	1101	$\{0,1,3\}$
14	$10\underline{0}1$	1110	$\{0,3\}$
15	$100\underline{0}$	1111	$\{3\}$

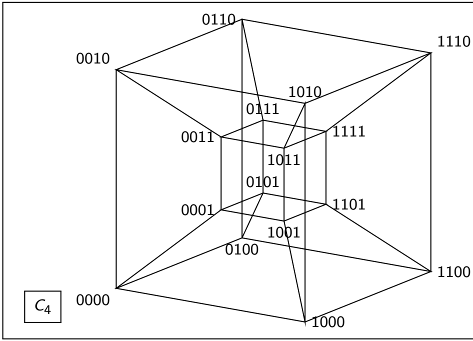
Hypercubes

<u>n-cube</u>: C_n défini par $V = \{0,1\}^n = \{\epsilon_1\epsilon_2\ldots\epsilon_n \mid \epsilon_i \in \{0,1\}\}$,
$E = \{\{\underline{\epsilon}, \underline{\epsilon}'\} \mid d(\underline{\epsilon}, \underline{\epsilon}') = 1\}$.



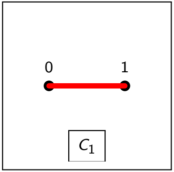


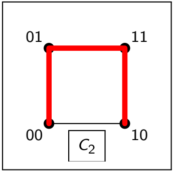


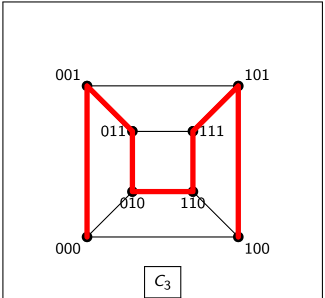


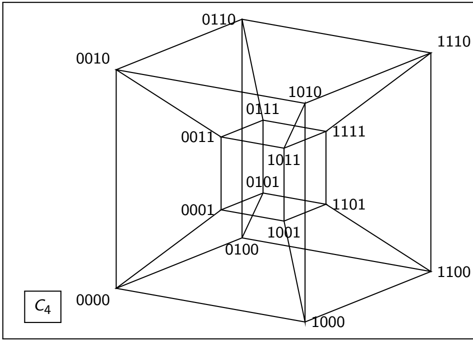
Hypercubes

<u>n-cube</u>: C_n défini par $V = \{0,1\}^n = \{\epsilon_1\epsilon_2\ldots\epsilon_n \mid \epsilon_i \in \{0,1\}\}$,
$E = \{\{\underline{\epsilon}, \underline{\epsilon}'\} \mid d(\underline{\epsilon}, \underline{\epsilon}') = 1\}$.



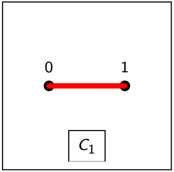


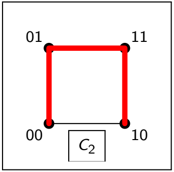


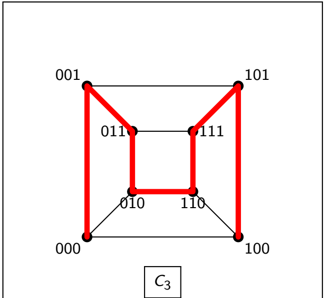


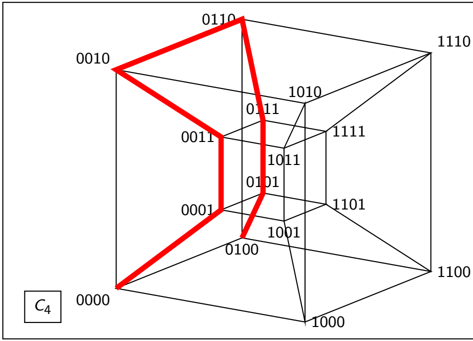
Hypercubes

$\underline{n\text{-cube}}$: C_n défini par $V = \{0,1\}^n = \{\epsilon_1\epsilon_2\ldots\epsilon_n \mid \epsilon_i \in \{0,1\}\}$,
$E = \{\{\underline{\epsilon}, \underline{\epsilon}'\} \mid d(\underline{\epsilon}, \underline{\epsilon}') = 1\}$.



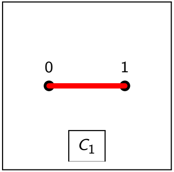


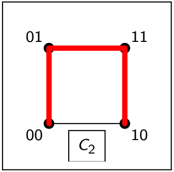


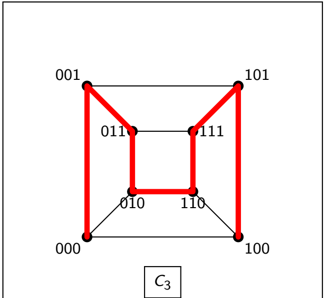


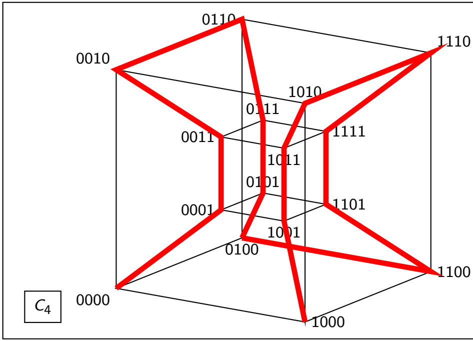
Hypercubes

<u>n-cube</u>: C_n défini par $V = \{0,1\}^n = \{\epsilon_1\epsilon_2\ldots\epsilon_n \mid \epsilon_i \in \{0,1\}\}$,
$E = \{\{\underline{\epsilon}, \underline{\epsilon}'\} \mid d(\underline{\epsilon}, \underline{\epsilon}') = 1\}$.

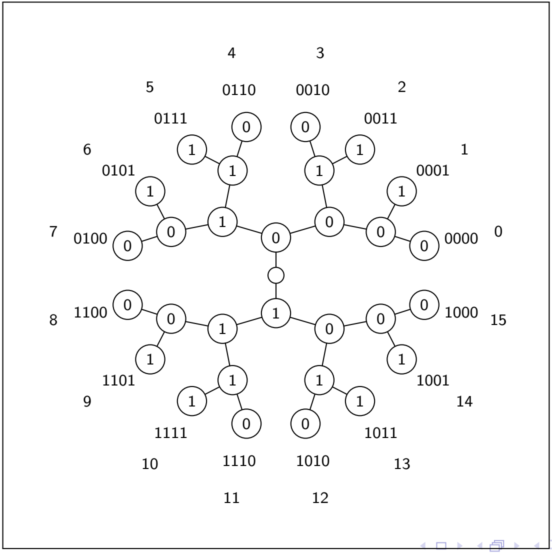








Codage angulaire



Le permanent

Le déterminant:

$$\det(A) = \sum_{\sigma \in \mathfrak{S}_n} \mathrm{s}(\sigma) \prod_{i=1}^{n} a_{i\sigma(i)}$$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{array}{r} a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \\ -a_{11}a_{23}a_{32} - a_{13}a_{22}a_{31} - a_{12}a_{21}a_{33} \end{array}$$

Le permanent:

$$\mathrm{perm}(A) = \sum_{\sigma \in \mathfrak{S}_n} \prod_{i=1}^{n} a_{i\sigma(i)}$$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}_p = \begin{array}{r} a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \\ +a_{11}a_{23}a_{32} + a_{13}a_{22}a_{31} + a_{12}a_{21}a_{33} \end{array}$$

Le permanent

Formule de Ryser:

$$\mathrm{perm}(A) = \sum_{S \subset [1,n]} (-1)^{n-|S|} \prod_{i=1}^{n} \sum_{j \in S} a_{ij}.$$

$$\begin{aligned} \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}_p =& a_{11}a_{22} + a_{12}a_{21} \\ =& 0 - a_{11}a_{21} - a_{12}a_{22} + (a_{11} + a_{12})(a_{21} + a_{22}) \end{aligned}$$

On passe de $O(2^{n-1}n^2)$ à $O(2^{n-1}n)$

Une applet

https://simonhung.github.io/NineLinkedRings/nineLinkedRings.html