

Feynman-Kac particle models and their applications to stochastic engineering

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Outline

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 - Some "heuristic-metaheuristic" like algorithms
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 - A nonlinear approach
 - Mean field particle methods
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 - A direct conditioning approach
 - Static models and Boltzmann-Gibbs flows
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Stochastic engineering \rightsquigarrow Conditional & Boltzmann-Gibbs' measures

- **Filtering:** Signal-Observation (X_t, Y_t) [Radar, Sonar, GPS, ...]

$$\eta_t = \text{Law}((X_0, \dots, X_t) \mid (Y_0, \dots, Y_t))$$

- **Rare events:** [Overflows, ruin processes, epidemic propagations, ...]

$$\eta_t = \text{Law}((X_0, \dots, X_t) \mid \text{Rare event}) \quad \text{and} \quad \mathbb{P}(X \in \text{Rare event})$$

- **Molecular simulation:** [ground state energies, directed polymers...]

$$\eta_t := \text{Boltzmann-Gibbs} \quad \text{or} \quad \eta_t = \text{Law}(X_t \mid \text{non absorption})$$

- **Combinatorial counting:** [\mathcal{A} random walks, unif. sampling]

- **Global optimization, hidden Markov problems**

$$\eta_t = \frac{1}{Z_t} e^{-\beta_t V(x)} \lambda(dx) \quad \text{or} \quad \eta_t = \frac{1}{Z_t} p_\theta(y_t) \lambda(d\theta)$$

Since the 50's \rightsquigarrow \uparrow bio-inspired sampling strategies

- **Filtering:** Particle, spawning, switching, bootstrap, condensed, ensemble, auxiliary, importance resampling,...filters.
- **Rare events:** Multi splitting, restart, subset sampling,...
- **Molecular simulation:** quantum and diffusion Monte-Carlo methods, matrix reconfigurations, reptation, pruning enrichment, go-with-the-winner, ...
- **Combinatorial counting:** cloning, pruning, branch and cut. ...
- **Global optimization, hidden Markov problems:** genetic, evolutionary alg., tabu search, survival the fittest, population Monte Carlo,...

~ Same natural dynamic heuristic:

- 1 *Explore randomly the solution space.*
- 2 *Duplicate nice proposals and stop-kill the other ones.*

A pair of "conjectures"

- 1 Previous engineering problems
 - ⊂ A single **Feynman-Kac** representation model
- 2 Previous "heuristic-metaheuristics"
 - ⊂ A single **mean field particle** interpretation model.

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Immediate answer :

- 1 **Both are true**
- 2 **Proof:** *Partly today/....*

Since 90's \rightsquigarrow Series of joints works with: *Arnaud Doucet, Alice Guionnet, Jean Jacod, Laurent Miclo, and many others.*

R.P. Feynman [Princeton Ph.D. in the 40's]

- **Feynman-Kac measures:** Markov-Potential (X_n, G_n) on E_n

$$d\eta_n = \frac{1}{\mathcal{Z}_n} \left\{ \prod_{0 \leq p < n} G_p(X_p) \right\} d\mathbb{P}_n^X$$

- **Weak representation:**

$[f_n$ test funct. on E_n , up to a state enlargement $X_n = (X'_0, \dots, X'_n)]$

$$\eta_n(f_n) = \frac{\gamma_n(f_n)}{\gamma_n(\mathbf{1})} \quad \text{with} \quad \gamma_n(f_n) = \mathbb{E} \left(f_n(X_n) \prod_{0 \leq p < n} G_p(X_p) \right)$$

- **A Key multiplicative formula:** (\rightsquigarrow Unbias estimation)

$$\mathcal{Z}_n = \mathbb{E} \left(\prod_{0 \leq p < n} G_p(X_p) \right) = \prod_{0 \leq p < n} \eta_p(G_p)$$

Some "wrong" approximation ideas

- "Pure" weighted Monte Carlo methods : X^i iid copies of X

$$\frac{1}{N} \sum_{i=1}^N f_n(X_n^i) \left\{ \prod_{0 \leq p < n} G_p(X_p^i) \right\} \simeq \mathbb{E} \left(f_n(X_n) \prod_{0 \leq p < n} G_p(X_p) \right)$$

↪ bad grids $X^i \oplus$ degenerate weights (running ex $G_n = 1_A$).

- Uncorrelated MCMC for each target measure η_n (\uparrow complex.).
- "Pure" branching ↪ critical random population sizes

$$G_n(x) = \mathbb{E}(g_n(x)) \quad \text{with} \quad g_n(x) \text{ r.v. } \in \mathbb{N}$$

- Harmonic/(Gaussian+linearisation) approximations.
- $G.M(H) \propto H \rightsquigarrow G \propto H/M(H) \rightsquigarrow H$ -process X^H (unknown).

Nonlinear distribution flows

Evolution equation: $[\eta_n \in \mathcal{P}(E_n)$ probability measures \uparrow complexity].

$$\eta_{n+1} = \Phi_{n+1}(\eta_n) = \Psi_{G_n}(\eta_n)M_{n+1}$$

With only 2 transformations:

- **X-Free Markov transport eq. :** $[M_n(x_{n-1}, dx_n)$ from E_{n-1} into $E_n]$

$$(\eta_{n-1}M_n)(dx_n) := \int_{E_{n-1}} \eta_{n-1}(dx_{n-1}) M_n(x_{n-1}, dx_n)$$

- **Bayes-Boltzmann-Gibbs transformation :**

$$\Psi_{G_n}(\eta_n)(dx_n) := \frac{1}{\eta_n(G_n)} G_n(x_n) \eta_n(dx_n)$$

Nonlinear distribution flows

- **Nonlinear Markov models** : always $\exists K_{n,\eta}(x, dy)$ Markov s.t.

$$\eta_n = \Phi_n(\eta_{n-1}) = \eta_{n-1} K_{n,\eta_{n-1}} = \text{Law}(\bar{X}_n)$$

i.e. :

$$\mathbb{P}(\bar{X}_n \in dx_n \mid \bar{X}_{n-1}) = K_{n,\eta_{n-1}}(\bar{X}_{n-1}, dx_n)$$

Mean field particle interpretation

- **Markov chain** $\xi_n = (\xi_n^1, \dots, \xi_n^N) \in E_n^N$ s.t.

$$\eta_n^N := \frac{1}{N} \sum_{1 \leq i \leq N} \delta_{\xi_n^i} \underset{N \uparrow \infty}{\simeq} \eta_n$$

- Particle approximation transitions ($\forall 1 \leq i \leq N$)

$$\xi_{n-1}^i \rightsquigarrow \xi_n^i \sim K_{n,\eta_{n-1}^N}(\xi_{n-1}^i, dx_n)$$

Discrete generation mean field particle model

Schematic picture : $\xi_n \in E_n^N \rightsquigarrow \xi_{n+1} \in E_{n+1}^N$

$$\begin{array}{ccc}
 \xi_n^1 & \xrightarrow{K_{n+1, \eta_n^N}} & \xi_{n+1}^1 \\
 \vdots & & \vdots \\
 \xi_n^i & \longrightarrow & \xi_{n+1}^i \\
 \vdots & & \vdots \\
 \xi_n^N & \longrightarrow & \xi_{n+1}^N
 \end{array}$$

Rationale :

$$\begin{aligned}
 \eta_n^N \simeq_{N \uparrow \infty} \eta_n &\implies K_{n+1, \eta_n^N} \simeq_{N \uparrow \infty} K_{n+1, \eta_n} \\
 &\implies \xi_n^i \text{ almost iid copies of } \bar{X}_n
 \end{aligned}$$

Advantages

- Mean field model = **Stoch. linearization/perturbation tech.** :

$$\eta_n^N = \Phi_n(\eta_{n-1}^N) + \frac{1}{\sqrt{N}} W_n^N$$

with $W_n^N \simeq W_n$ independent and centered Gauss field.

- $\eta_n = \Phi_n(\eta_{n-1})$ stable \Rightarrow local errors do not propagate

\Rightarrow **uniform control of errors w.r.t. the time parameter**

- "No need" to study the cv of equilibrium of MCMC models.
- Adaptive stochastic grid approximations
- Take advantage of the nonlinearity of the system to define beneficial interactions. Non intrusive methods.
- Natural and easy to implement, etc.

Mean field particle methods

"Intuitive picture" \rightsquigarrow nonlinear sg : $\eta_n = \Phi_n(\eta_{n-1}) = \Phi_{p,n}(\eta_p) = \eta_n$

Local errors

$$W_n^N := \sqrt{N} \left[\eta_n^N - \Phi_n \left(\eta_{n-1}^N \right) \right] \simeq W_n \perp \text{Gaussian field}$$

Local transport formulation :

$$\begin{array}{ccccccc}
 \eta_0 & \rightarrow & \eta_1 = \Phi_1(\eta_0) & \rightarrow & \eta_2 = \Phi_{0,2}(\eta_0) & \rightarrow & \dots & \rightarrow & \Phi_{0,n}(\eta_0) \\
 \downarrow & & & & & & & & \\
 \eta_0^N & \rightarrow & \Phi_1(\eta_0^N) & \rightarrow & \Phi_{0,2}(\eta_0^N) & \rightarrow & \dots & \rightarrow & \Phi_{0,n}(\eta_0^N) \\
 & & \downarrow & & & & & & \\
 & & \eta_1^N & \rightarrow & \Phi_2(\eta_1^N) & \rightarrow & \dots & \rightarrow & \Phi_{1,n}(\eta_1^N) \\
 & & & & \downarrow & & & & \\
 & & & & \eta_2^N & \rightarrow & \dots & \rightarrow & \Phi_{2,n}(\eta_2^N) \\
 & & & & & & & & \vdots \\
 & & & & & & \downarrow & & \\
 & & & & & & \eta_{n-1}^N & \rightarrow & \Phi_n(\eta_{n-1}^N) \\
 & & & & & & & & \downarrow \\
 & & & & & & & & \eta_n^N
 \end{array}$$

\rightsquigarrow Key decomposition formula : [Propagation of local errors]+[works for any approximating scheme]

$$\begin{aligned}
 \eta_n^N - \eta_n &= \sum_{q=0}^n [\Phi_{q,n}(\eta_q^N) - \Phi_{q,n}(\Phi_q(\eta_{q-1}^N))] \\
 &\simeq \frac{1}{\sqrt{N}} \sum_{q=0}^n W_q^N D_{q,n} \quad \text{first order decomp. } \Phi_{p,n}(\eta) - \Phi_{p,n}(\mu) \simeq (\eta - \mu)D_{p,n} + (\eta - \mu)^{\otimes 2} \dots
 \end{aligned}$$

$$\Rightarrow \text{Example Unif. estimates + A 2 lines proof CLT : } \sqrt{N} \left[\eta_n^N - \eta_n \right] \simeq \sum_{q=0}^n W_q D_{q,n}$$

Some Theoretical results : TCL, PGD, PDM, ... (n, N) :

- McKean particle measure

$$\frac{1}{N} \sum_{i=1}^N \delta_{(\xi_0^i, \dots, \xi_n^i)} \simeq_N \text{Law}(\bar{X}_0, \dots, \bar{X}_n) \ \& \ \eta_n^N = \frac{1}{N} \sum_{i=1}^N \delta_{\xi_n^i} \simeq_N \eta_n$$

- Empirical Processes : $\sup_{n \geq 0} \sup_{N \geq 1} \sqrt{N} \mathbb{E}(\|\eta_n^N - \eta_n\|_{\mathcal{F}_n}^p) < \infty$
- Uniform concentration inequalities :

$$\sup_{n \geq 0} \mathbb{P}(|\eta_n^N(f_n) - \eta_n(f_n)| > \epsilon) \leq c \exp\{- (N\epsilon^2)/(2\sigma^2)\}$$

- Propagations of chaos : $\mathbb{P}_{n,q}^N := \text{Law}(\xi_n^1, \dots, \xi_n^q)$

$$\mathbb{P}_{n,q}^N \simeq \eta_n^{\otimes q} + \frac{1}{N} \partial^1 \mathbb{P}_{n,q} + \dots + \frac{1}{N^k} \partial^k \mathbb{P}_{n,q} + \frac{1}{N^{k+1}} \partial^{k+1} \mathbb{P}_{n,q}^N$$

with $\sup_{N \geq 1} \|\partial^{k+1} \mathbb{P}_{n,q}^N\|_{\text{tv}} < \infty$ & $\sup_{n \geq 0} \|\partial^1 \mathbb{P}_{n,q}\|_{\text{tv}} \leq c q^2$.

Ex.: Feynman-Kac distribution flows

- **FK-Nonlinear Markov models :**

$\epsilon_n = \epsilon_n(\eta_n) \geq 0$ s.t. η_n -a.e. $\epsilon_n G_n \in [0, 1]$ ($\epsilon_n = 0$ not excluded)

$$K_{n+1, \eta_n}(x, dz) = \int S_{n, \eta_n}(x, dy) M_{n+1}(y, dz)$$

$$S_{n, \eta_n}(x, dy) := \epsilon_n G_n(x) \delta_x(dy) + (1 - \epsilon_n G_n(x)) \Psi_{G_n}(\eta_n)(dy)$$

- **Mean field genetic type particle model :**

$$\xi_n^i \in E_n \xrightarrow{\text{accept/reject/selection}} \widehat{\xi}_n^i \in E_n \xrightarrow{\text{proposal/mutation}} \xi_{n+1}^i \in E_{n+1}$$

- **Running ex. : $G_n = 1_A \rightsquigarrow$ killing with uniform replacement.**

Mean field genetic type particle model :

$$\begin{array}{c} \xi_n^1 \\ \vdots \\ \xi_n^i \\ \vdots \\ \xi_n^N \end{array} \Bigg] \xrightarrow{S_{n,\eta_n^N}} \begin{array}{c} \widehat{\xi}_n^1 \\ \vdots \\ \widehat{\xi}_n^i \\ \vdots \\ \widehat{\xi}_n^N \end{array} \begin{array}{c} \xrightarrow{M_{n+1}} \\ \xrightarrow{\quad\quad\quad} \\ \xrightarrow{\quad\quad\quad} \\ \xrightarrow{\quad\quad\quad} \end{array} \begin{array}{c} \xi_{n+1}^1 \\ \vdots \\ \xi_{n+1}^i \\ \vdots \\ \xi_{n+1}^N \end{array} \Bigg]$$

Accept/Reject/Selection transition :

$$S_{n,\eta_n^N}(\xi_n^i, dx)$$

$$:= \epsilon_n G_n(\xi_n^i) \delta_{\xi_n^i}(dx) + (1 - \epsilon_n G_n(\xi_n^i)) \sum_{j=1}^N \frac{G_n(\xi_n^j)}{\sum_{k=1}^N G_n(\xi_n^k)} \delta_{\xi_n^j}(dx)$$

Running Ex. : $G_n = 1_A \rightsquigarrow G_n(\xi_n^i) = 1_A(\xi_n^i)$

Path space models

- $X_n = (X'_0, \dots, X'_n) \rightsquigarrow$ **genealogical tree/ancestral lines**

$$\eta_n^N := \frac{1}{N} \sum_{1 \leq i \leq N} \delta_{\xi_n^i} = \frac{1}{N} \sum_{1 \leq i \leq N} \delta_{(\xi_{0,n}^i, \xi_{1,n}^i, \dots, \xi_{n,n}^i)} \simeq_{N \uparrow \infty} \eta_n$$

- **Unbias particle approximations :**

$$\gamma_n^N(1) = \prod_{0 \leq p < n} \eta_p^N(G_p) \simeq_{N \uparrow \infty} \gamma_n(1) = \prod_{0 \leq p < n} \eta_p(G_p)$$

- *Running example* $G_n = 1_A :$

$$\Rightarrow \gamma_n^N(1) = \prod_{0 \leq p < n} (\text{success \% at } p)$$

Feynman-Kac particle sampling recipes

Nonlinear Feynman-Kac type flow $\sim (G_n, M_n)$

$$\eta_n = \Phi_n(\eta_{n-1}) = \Psi_{G_{n-1}}(\eta_{n-1})M_n$$

- Interacting stochastic algorithm

accept/reject/select/branch/prune/clone/spawn/enrich $\rightsquigarrow G_n$
 exploration/proposition/mutation/prediction $\rightsquigarrow M_n$

- Normalizing constants \rightsquigarrow key multiplicative formula.
- Path space models \rightsquigarrow path-particles=ancestral lines

Occupation meas. of genealogical trees $\simeq \eta_n \in$ path-space

- Tuning parameters: $(G_n, M_n) \sim$ change of ref. measures, path/excursion spaces, selection periods, weights interpretations,...

A direct conditioning approach

- **Filtering models:** Signal-Observation likelihood functions (X_n, G_n)

$$\eta_n = \text{Law}((X_0, \dots, X_n) \mid (Y_0, \dots, Y_n))$$

- **Rare events:**

- Confinements potentials: $G_n = 1_{A_n}$

$$\eta_n = \text{Law}((X_0, \dots, X_n) \mid X_0 \in A_0, \dots, X_n \in A_n)$$

$$\mathcal{Z}_n = \mathbb{P}(X_0 \in A_0, \dots, X_n \in A_n)$$

- Twisted measures $\sim \mathbb{P}(V_n(X_n) \geq a)$?

$$\mathbb{E}(f_n(X_n) e^{\lambda V_n(X_n)}) = \mathbb{E} \left(f_n(X_n) \prod_{0 \leq p \leq n} e^{\lambda(V_p(X_p) - V_{p-1}(X_{p-1}))} \right)$$

- **Hitting B before C :**

- *Multi-level decomposition* $B_0 \supset B_1 \supset \dots \supset B_m$, $B_0 \cap C = \emptyset$.
- *Inter-level excursions* :

$$T_n = \inf \{p \geq T_{n-1} : Y_p \in B_n \cup C\}$$

- *Level excursions and level detection potentials:*

$$X_n = (Y_p ; T_{n-1} \leq p \leq T_n) \quad \text{and} \quad G_n(X_n) = 1_{B_n}(Y_{T_n})$$



$$\mathbb{P}(Y \text{ hits } B_m \text{ before } C) = \mathbb{E} \left(\prod_{1 \leq p \leq m} G_p(X_p) \right)$$

$$\mathbb{E}(f(Y_0, \dots, Y_{T_m}) 1_{B_m}(Y_{T_m})) = \mathbb{E} \left(f(X_0, \dots, X_m) \prod_{1 \leq p \leq m} G_p(X_p) \right)$$

- **Markov processes with fixed terminal values**

- π "target type" measure + (K, L) pair Markov transitions

$$\text{Metropolis potential } G(x_1, x_2) = \frac{\pi(dx_2)L(x_2, dx_1)}{\pi(dx_1)K(x_1, dx_2)}$$

- [A Time reversal formula] :

$$\begin{aligned} & \mathbb{E}_\pi^L(f_n(X_n, X_{n-1}, \dots, X_0) | X_n = x) \\ &= \frac{\mathbb{E}_x^K(f_n(X_0, X_1, \dots, X_n) \{\prod_{0 \leq p < n} G(X_p, X_{p+1})\})}{\mathbb{E}_x^K(\{\prod_{0 \leq p < n} G(X_p, X_{p+1})\})} \end{aligned}$$

- **Non intersecting random walks & connectivity constants:**

$$X_n := (X'_0, \dots, X'_n) \quad \text{and} \quad G_n(X_n) = 1_{\notin \{X'_p, p < n\}}(X'_n)$$

$$\eta_n = \text{Law}((X'_0, \dots, X'_n) \mid \forall p < q < n \quad X'_p \neq X'_q)$$

- Molecular simulation \sim Particle absorption models

- X_n Markov $\in (E_n, \mathcal{E}_n)$ with transitions M_n , and $G_n : E_n \rightarrow [0, 1]$

$$Q_n(x, dy) = G_{n-1}(x) M_n(x, dy) \quad \text{sub-Markov operator}$$

- $\rightsquigarrow E_n^c = E_n \cup \{c\}$.

$$X_n^c \in E_n^c \xrightarrow{\text{absorption} \sim G_n} \widehat{X}_n^c \xrightarrow{\text{exploration} \sim M_n} X_{n+1}^c$$

With:

- **Absorption:** $\widehat{X}_n^c = X_n^c$, with proba $G(X_n^c)$; otherwise $\widehat{X}_n^c = c$.
- **Exploration:** elementary free explorations $X_n \rightsquigarrow X_{n+1}$

Feynman-Kac integral model

- $T = \inf \{n : \widehat{X}_n^c = c\}$ **absorption time** : $\forall f_n \in \mathcal{B}_b(E_n)$

$$\mathbb{P}(T \geq n) = \gamma_n(1) := \mathbb{E} \left(\prod_{0 \leq p < n} G(X_p) \right)$$

$$\mathbb{E}(f_n(X_n^c) ; (T \geq n)) = \gamma_n(f_n) := \mathbb{E} \left(f_n(X_n) \prod_{0 \leq p < n} G_p(X_p) \right)$$

- **Continuous time models** : $\Delta = \text{time step}$

$$(M, G) = (Id + \Delta L, e^{-V\Delta}) \implies Q \rightsquigarrow L^V := L - V$$

$\rightsquigarrow L$ -motions \oplus expo. clocks rate $V \oplus$ Uniform selection.

Spectral radius-Lyapunov exponents

- $Q(x, dy) = G(x)M(x, dy)$ sub-Markov operator on $\mathcal{B}_b(E)$
- **Normalized FK-model** : $\eta_n(f) = \gamma_n(f)/\gamma_n(1)$.

$$\mathbb{P}(T \geq n) = \mathbb{E} \left(\prod_{0 \leq p \leq n} G(X_p) \right) = \prod_{0 \leq p \leq n} \eta_p(G) \simeq e^{-\lambda n}$$

with $e^{-\lambda} \stackrel{M}{=}^{reg.} Q$ -top eigenvalue or

$$\begin{aligned} \lambda &= -\text{LogLyap}(Q) = \lim_{n \rightarrow \infty} -\frac{1}{n} \log \|Q^n\| \\ &= -\frac{1}{n} \log \mathbb{P}(T \geq n) = -\frac{1}{n} \sum_{0 \leq p \leq n} \log \eta_p(G) = -\log \eta_\infty(G) \end{aligned}$$

Feynman-Kac-Schrodinger ground states energies

M μ – reversible :

$$\Rightarrow \mathbb{E}(f(X_n^c) \mid T > n) \simeq \frac{\mu(H f)}{\mu(H)} \quad \text{with} \quad Q(H) = e^{-\lambda H}$$

Limiting FK-measures

$$\eta_n = \Phi(\eta_{n-1}) \xrightarrow{n \uparrow \infty} \eta_\infty \quad \text{with} \quad \frac{\eta_\infty(G f)}{\eta_\infty(G)} = \frac{\mu(H f)}{\mu(H)}$$

leadsto Particle approximations :

$$\lambda \simeq_{n, N \uparrow} \lambda_n^N := \frac{1}{n} \sum_{0 \leq p \leq n} \log \eta_p^N(G) \quad \text{and} \quad \eta_\infty \simeq_{n, N \uparrow} \eta_n^N$$

$\text{Law}((X_0^c, \dots, X_n^c) \mid (T \geq n)) \simeq$ Genealogical tree measures

Boltzmann-Gibbs measures

- X r.v. $\in (E, \mathcal{E})$ with $\mu = \text{Law}(X)$
- Target measures associated with $g_n : E \rightarrow \mathbb{R}_+$

$$\eta_n(dx) := \Psi_{g_n}(\mu)(dx) = \frac{1}{\mu(g_n)} g_n(x) \mu(dx)$$

Running examples :

$$g_n = 1_{A_n} \Rightarrow \eta_n(dx) \propto 1_{A_n}(x) \mu(dx)$$

$$g_n = e^{-\beta_n V} \Rightarrow \eta_n(dx) \propto e^{-\beta_n V(x)} \mu(dx)$$

$$g_n = \prod_{0 \leq p \leq n} h_p \Rightarrow \eta_n(dx) \propto \left\{ \prod_{0 \leq p \leq n} h_p(x) \right\} \mu(dx)$$

Applications : global optimization pb., tails distributions, hidden Markov chain models, etc.

Boltzmann-Gibbs distribution flows

- Target distribution flow : $\eta_n(dx) \propto g_n(x) \mu(dx)$
- Product hypothesis :

$$g_n = g_{n-1} \times G_{n-1} \implies \eta_n = \Psi_{G_{n-1}}(\eta_{n-1})$$

Running Examples:

$$\begin{aligned} g_n &= 1_{A_n} & \text{with } A_n \downarrow & \implies G_{n-1} = 1_{A_n} \\ g_n &= e^{-\beta_n V} & \text{with } \beta_n \uparrow & \implies G_{n-1} = e^{-(\beta_n - \beta_{n-1})V} \\ g_n &= \prod_{0 \leq p \leq n} h_p & & \implies G_{n-1} = h_n \end{aligned}$$

- **Problem** : $\eta_n = \Psi_{G_{n-1}}(\eta_{n-1}) = \text{unstable equation.}$

FK-stabilization

- Choose $M_n(x, dy)$ s.t. local fixed point eq. $\rightarrow \eta_n = \eta_n M_n$
(Metropolis, Gibbs,...)

- **Stable equation :**

$$\begin{aligned}g_n = g_{n-1} \times G_{n-1} &\implies \eta_n = \Psi_{G_{n-1}}(\eta_{n-1}) \\ &\implies \eta_n = \eta_n M_n = \Psi_{G_{n-1}}(\eta_{n-1}) M_n\end{aligned}$$

- **Feynman-Kac "dynamical" formulation (X_n Markov M_n)**

$$\int f(x) g_n(x) \mu(dx) \propto \mathbb{E} \left(f(X_n) \prod_{0 \leq p < n} G_p(X_p) \right)$$

- \rightsquigarrow **Interacting Metropolis/Gibbs/... stochastic algorithms.**

Interacting stochastic simulation algorithms

- **Mean field and Feynman-Kac particle models :**

- Feynman-Kac formulae. Genealogical and interacting particle systems, Springer (2004) \oplus Refs.
- joint work with L. Miclo. A Moran particle system approximation of Feynman-Kac formulae. *Stochastic Processes and their Applications*, Vol. 86, 193-216 (2000).
- joint work with L. Miclo. Branching and Interacting Particle Systems Approximations of Feynman-Kac Formulae. *Séminaire de Probabilités XXXIV, Lecture Notes in Mathematics*, Springer-Verlag Berlin, Vol. 1729, 1-145 (2000).

- **Sequential Monte Carlo models :**

- joint work with Doucet A., Jasra A. Sequential Monte Carlo Samplers. *JRSS B* (2006).
- joint work with A. Doucet. On a class of genealogical and interacting Metropolis models. *Sém. de Proba.* 37 (2003).

Particle rare event simulation algorithms

- **Twisted Feynman-Kac measures**

- joint work with J. Garnier. Genealogical Particle Analysis of Rare events. *Annals of Applied Probab.*, 15-4 (2005).
- joint work with J. Garnier. Simulations of rare events in fiber optics by interacting particle systems. *Optics Communications*, Vol. 267 (2006).

- **Multi splitting excursion models**

- joint work with P. Lezaud. Branching and interacting particle interpretation of rare event proba.. *Stochastic Hybrid Systems : Theory and Safety Critical Applications*, eds. H. Blom and J. Lygeros. Springer (2006).
- joint work with F. Cerou, Le Gland F., Lezaud P. *Genealogical Models in Entrance Times Rare Event Analysis*, *Alea*, Vol. I, (2006).

Particle rare event simulation algorithms

- **Particle absorption models**

- joint work with L. Miclo. Particle Approximations of Lyapunov Exponents Connected to Schrodinger Operators and Feynman-Kac Semigroups. ESAIM Probability & Statistics, vol. 7, pp. 169-207 (2003).
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A new generation of i-MCMC algorithms

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