ERRATA & NOTES

Theory and Applications of Abstract Semilinear Cauchy Problems
by Pierre Magal and Shigui Ruan

- p. xii replace $M:\mathbb{R}\times L^1((0,+\infty),\mathbb{R}^n)\to L^1((0,+\infty),\mathbb{R}^n)$ by $M:\mathbb{R}\times L^p((0,+\infty),\mathbb{R}^n)\to L^p((0,+\infty),\mathbb{R}^n)$.
- p. xii replace $B:\mathbb{R}\times L^1((0,+\infty),\mathbb{R}^n)\to\mathbb{R}^n$ by $B:\mathbb{R}\times L^p((0,+\infty),\mathbb{R}^n)\to\mathbb{R}^n$.
- p. 21 In Assumption 1.1.27 $(A+\partial_x F(\mu,0))_0$ should be replaced by $(A+\partial_x F(\mu,0))$.
- p. 22 In the Hopf bifurcation Theorem 1.1.28 $\varepsilon\to x_\varepsilon$ from $(0,\varepsilon^*)$ into $\mathbb{R}^n$ should be renamed $\varepsilon\to x_{0\varepsilon}$. Then the initial value $x_\varepsilon(0)=x_0$ should be replaced by $x_\varepsilon(0)=x_{0\varepsilon}$ (twice in the text and in the equation).
- p. 92 l-8v $C_b([0,1),\mathbb{R})$ should be replaced by BC$([0,1),\mathbb{R})$ (Bounded Continuous).
- p. 95 In Example 2.6.7. the notation $UBC(\mathbb{R},\mathbb{R})$ should be replaced by BUC$(\mathbb{R},\mathbb{R})$ (Bounded Uniformly Continuous).
- p. 113 l+4 "scalar product" should be replaced by "duality product".
- p. 119 l+8 $dr\,dl$ should be $dl\,dr$
- p. 119 l+9 $dr\,dl$ should be dr.
- p. 119 l+11 The equation should be

$$(S_A * f)(t) = \int_0^t S_A(s)f(0)ds + \int_0^t \int_0^{t-l} S_A(r)f'(l)dr\,dl.$$

- p.135 In the Proposition 3.7.1. the last formula should be

$$\|\varphi\|_{L^p(J,Z)} = \sup_{\substack{\psi\in C_c^\infty(J,Z^*)\\ \|\psi\|_{L^q(J,Z^*)}\leq 1}} \int_J \psi(s)\left(\varphi(s)\right)ds.$$

- p.153 l+1 "Corollary 2.2.13" should be replaced by "Corollary 2.2.15"

- p.158 l+6 $(\lambda I - \mathcal{A})^{-1})$ should be replaced by $(\lambda I - \mathcal{A})^{-1}$
- p.164 Kellermann and Hiber should be replaced by Kellermann and Hieber
- p.189 There is a confusion between the index k used for the space E_k and T^k used in the part (b) of the proof of Theorem 4.3.16. There must be two different indexes. The proof reads as follows.

  Proof. (b) We prove $\dim(E_{k_0}) < +\infty$ by induction. Clearly $E_0 = \{0\}$. Thus,
  $$\dim(E_0) = 0.$$
  Assume that $\dim(E_k) < +\infty$. Let $u \in B_{E_{k+1}}(0,1)$, then from part (a) of the proof we know that there exists $v \in E_k$ such that
  $$Tu = u - v.$$
  We have
  $$\|v\| \leq (1 + \|T\|) =: \delta$$
  and
  $$T^m(u) = u - \sum_{l=0}^{m-1} T^l(v) \Leftrightarrow u = T^m(u) + \sum_{l=0}^{m-1} T^l(v).$$
  Hence
  $$\kappa\left(B_{E_{k+1}}(0,1)\right) \leq \kappa\left[T^m(B_X(0,1)) + B_{E_k}(0,\delta) + TB_{E_k}(0,\delta) + ... + T^{m-1}B_{E_k}(0,\delta)\right],$$
  and, since $\dim(E_k) < +\infty$, we obtain
  $$\kappa\left(B_{E_{k+1}}(0,1)\right) \leq \kappa\left(T^m(B_X(0,1))\right), \ \forall m \geq 1.$$
  When m goes to $+\infty$, since $r_{\text{ess}}(T) < 1$, it follows that $\kappa(T^m(B_X(0,1))) \to 0$. Thus,
  $$\kappa\left(B_{E_{k+1}}(0,1)\right) = 0.$$
  It implies that $\overline{B_{E_{k+1}}(0,1)}$ is compact. But $(I-T)^{k+1}$ is bounded, we deduce that $E_{k+1} = \mathcal{N}((I-T)^{k+1})$ is closed, so is $B_{E_{k+1}}(0,1)$. Hence, $B_{E_{k+1}}(0,1)$ is compact. Now by applying the Riesz's theorem we obtain that $\dim(E_{k+1}) < +\infty$. ∎

- p.189 l-6 $X_n = \mathcal{R}((I-T)^n X)$, should be replaced by $X_n = \mathcal{R}((I-T)^n)$.
- p.189 l-4 $f \in \mathcal{R}((I-T)X_k)$ should be replaced by $f \in \mathcal{R}((I-T)|_{X_k})$.
- p.204 l-8 In Lemma 4.5.1. we mean $\forall \lambda \in \rho(A_Y)$.
- p.222 l-11 In the proof of Lemma 5.2.3 (Uniqueness) $\delta(t)$ should be replaced by $\delta(t-t_0)$. Therefore the estimation should be
  $$\|u(t) - v(t)\| \leq \delta(t-t_0) K(\tau + s, \xi) \sup_{l \in [t_0, t_0+t]} \|u(l) - v(l)\|.$$

- p.222-223 The statement of Lemma 5.2.4 and its proof is not correct. In $\delta(\gamma(\tau,\beta,\xi))$ we should drop some $\delta(.)$ which was not there in the original result (see Lemma 5.4. in [1]). The original result and its proof should be the following.

  Lemma 0.1 (Local Existence) *Let Assumptions 5.1.1, 5.1.2, and 5.2.1 be satisfied. Then for each $\tau > 0$, each $\beta > 0$, and each $\xi > 0$, there exists $\gamma(\tau,\beta,\xi) \in (0,\tau_0]$ such that for each $s \in [0,\tau]$ and each $x \in X_0$ with $|x| \le \xi$, equation (5.1.1) has a unique integrated solution $U(.,s)x \in C([s,s+\gamma(\tau,\beta,\xi)], X_0)$ which satisfies*

  $$|U(t,s)x| \le (1+\beta)\xi,\ \forall t \in [s,s+\gamma(\tau,\beta,\xi)].$$

  Proof. Let $s \in [0,\tau]$ and $x \in X_0$ with $\|x\| \le \xi$ be fixed. Let $\gamma(\tau,\beta,\xi) \in (0,\tau_0]$ such that

  $$\delta(\gamma(\tau,\beta,\xi))\, M\left[\widehat{\xi}_{\tau+\tau_0} + (1+\beta)\,\xi K(\tau+\tau_0,(1+\beta)\,\xi)\right] \le \beta\xi$$

  with $\widehat{\xi}_\alpha = \sup_{s\in[0,\alpha]}\|F(s,0)\|, \forall \alpha \ge 0$. Set

  $E = \{u \in C([s,s+\gamma(\tau,\beta,\xi)], X_0) : |u(t)| \le (1+\beta)\,\xi, \forall t \in [s,s+\gamma(\tau,\beta,\xi)]\}.$

  Consider the map $\Phi_{x,s} : C([s,s+\gamma(\tau,\beta,\xi)], X_0) \to C([s,s+\gamma(\tau,\beta,\xi)], X_0)$ defined for each $t \in [s,s+\gamma(\tau,p,C)]$ by

  $$\Phi_{x,s}(u)(t) = T_{A_0}(t-s)x + \frac{d}{dt}(S_A * F(.+s,u(.+s)))(t-s).$$

  We have $\forall u \in E$ that (using (5.2.1) repeatedly)

  $$\begin{aligned}
  |\Phi_{x,s}(u)(t)| &\le \xi + M\left\|\frac{d}{dt}(S_A * F(.+s,u(.+s)))(t-s)\right\| \\
  &\le \xi + M\delta(\gamma(\tau,\beta,\xi)) \sup_{t\in[s,s+\gamma(\tau,\beta,\xi)]}\|F(t,u(t))\| \\
  &\le \xi + M\delta(\gamma(\tau,\beta,\xi))\left[\widehat{\xi}_\alpha + K(\tau+\tau_0,(1+\beta)\,\xi)\sup_{t\in[s,s+\gamma(\tau,\beta,\xi)]}|u(t)|\right] \\
  &\le (1+\beta)\,\xi.
  \end{aligned}$$

  Hence, $\Phi_{x,s}(E) \subset E$. Moreover, for all $u, v \in E$, we have (again using (5.2.1))

  $$\begin{aligned}
  &|\Phi_{x,s}(u)(t) - \Phi_{x,s}(v)(t)| \\
  &\quad\le M\delta(\gamma(\tau,\beta,\xi))\, K(\tau+\tau_0,(1+\beta)\,\xi) \sup_{t\in[s,s+\gamma(\tau,\beta,\xi)]}|u(t)-v(t)| \\
  &\quad\le \frac{K(\tau+\tau_0,(1+\beta)\,\xi)\beta\xi}{1+\widehat{\xi}_\alpha + K(\tau+\tau_0,(1+\beta)\,\xi)\,(1+\beta)\,\xi} \sup_{t\in[s,s+\gamma(\tau,\beta,\xi)]}|u(t)-v(t)| \\
  &\quad\le \frac{\beta}{1+\beta}\sup_{t\in[s,s+\gamma(\tau,\beta,\xi)]}|u(t)-v(t)|.
  \end{aligned}$$

[1] P. Magal, and S. Ruan (2007), On Integrated Semigroups and Age Structured Models in L^p Spaces, Differential and Integral Equations,2, 197-139.

Therefore, $\Phi_{x,s}$ is a $\left(\frac{\beta}{1+\beta}\right)$-contraction on E and the result follows. ■

- p.229 The last inequality of Corollary 5.3.4. is only true for $x \in X_{0+}$. So it should be

$$\|U(t,s)x\| \leq e^{\gamma(t-s)}\left[C_1\|x\| + C_2\right], \forall x \in X_{0+}.$$

- p. 259 l-1 and p. 260 l+1. The end of the proof of Theorem 6.1.10 (i) should be By projecting on X_{0h}, we obtain

$$\Pi_{0h} u_{x_c} = [K_s + K_u]\, \Phi_{\Pi_h F}\left(u_{x_c}\right),$$

so

$$\Psi(x_c) = [K_s + K_u]\, \Phi_{\Pi_h F}\left(u_{x_c}\right)(0) \tag{0.1}$$

and (i) follows.

- p. 262 l-2 (and p. 263 l+1)

$$\alpha_n := d\left(H_1(x_0(n), \overline{x}_1), H_1(x_0(n), \overline{x}_1)\right)$$

should be replaced by

$$\alpha_n := d\left(H_1(x_0(n), \overline{x}_1), H_1(\overline{x}_0, \overline{x}_1)\right).$$

- p.313 l-11 In the proof of Lemma 7.1.2 it should be

$$M_A := \sup_{t\geq 0}\left\| e^{(B-\omega_A I)t}\right\|_{\mathcal{L}(\mathbb{R}^n)}.$$