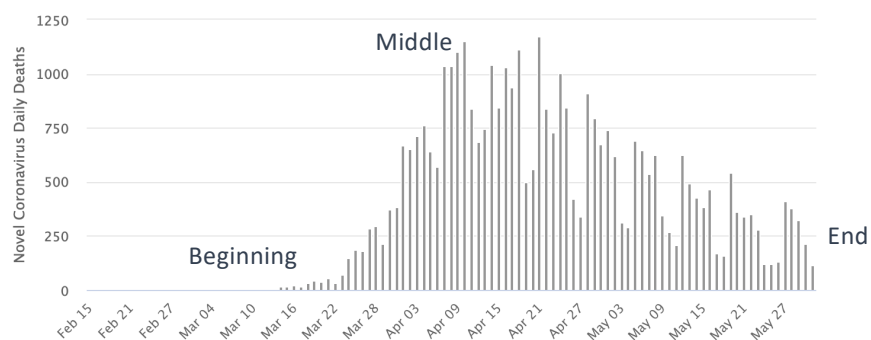


# Epidemic modelling at different stages of infectious disease outbreaks

Robin Thompson  
Junior Research Fellow  
Christ Church, University of Oxford



## Beginning

- Will initial cases lead to a major epidemic?
- Which interventions reduce the epidemic risk?

## Middle

- How effective are current interventions?
- Which interventions will minimise numbers of cases?

## End

- How should interventions be lifted?
- Is the epidemic over?

## Outline

### 1. Assessing the risk of a major epidemic

- Estimating the probability of a major epidemic
- Application to COVID-19
- More complex models: heterogeneity in reporting, age structure, time-dependence

### 2. Estimating changes in disease transmissibility

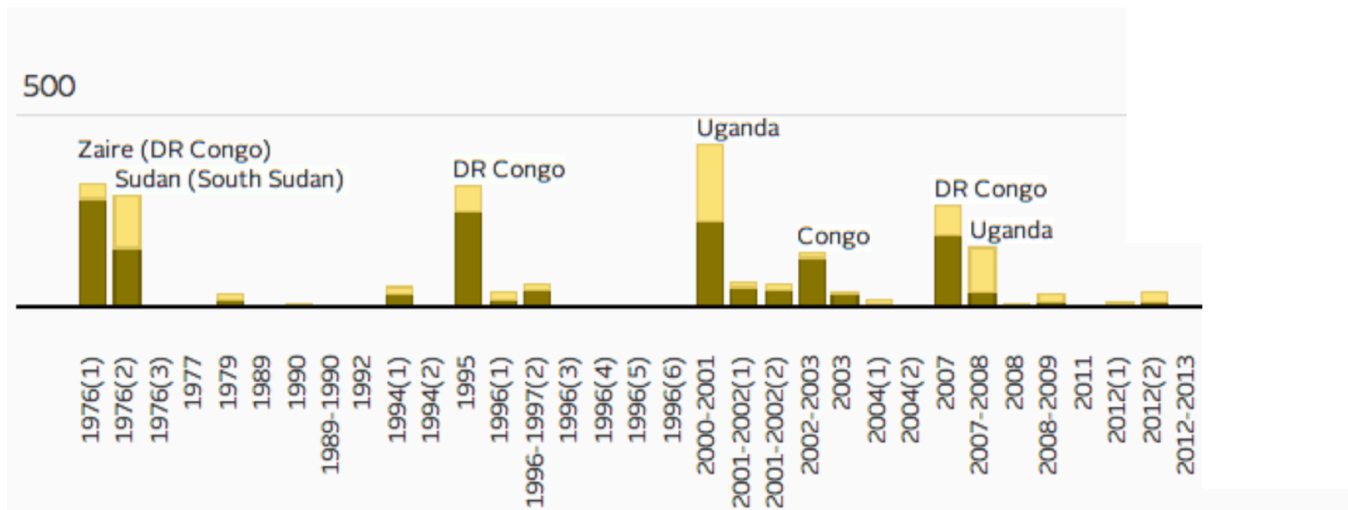
- Estimation of  $R_t$
- Extending the basic model
- Going forwards

## Assessing the risk of a major epidemic

When a pathogen first arrives in a new host population, will initial cases fade out, or will they lead to a major epidemic?

## Assessing the risk of a major epidemic

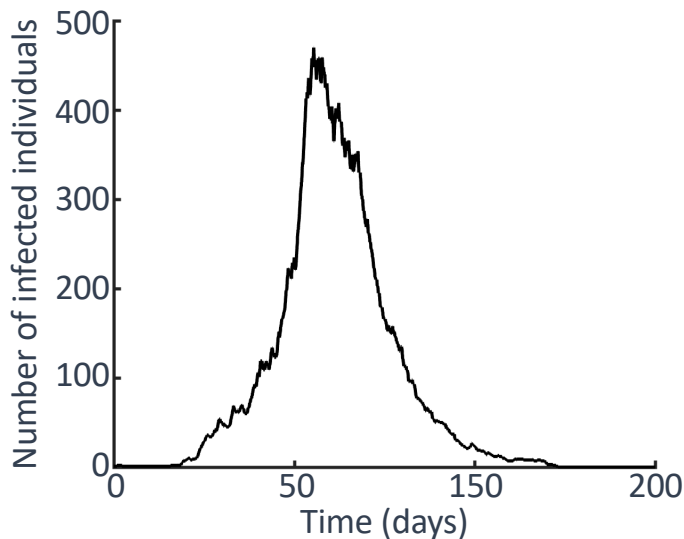
■ Died ■ Survived



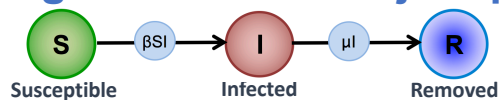
## Assessing the risk of a major epidemic



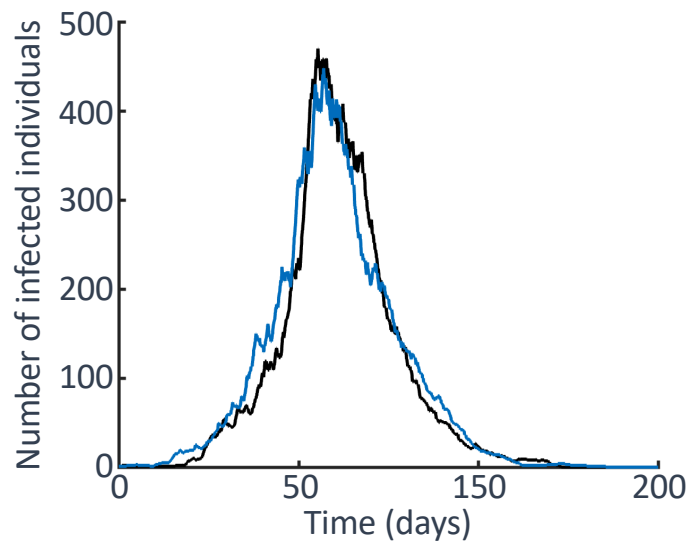
When a pathogen first arrives in a new population, there are two possibilities for what happens next



## Assessing the risk of a major epidemic



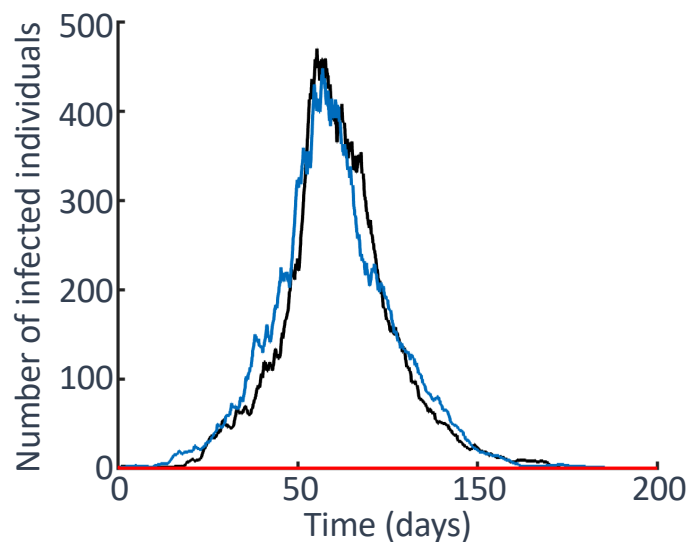
When a pathogen first arrives in a new population, there are two possibilities for what happens next



## Assessing the risk of a major epidemic

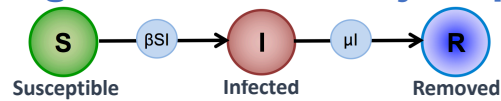


When a pathogen first arrives in a new population, there are two possibilities for what happens next





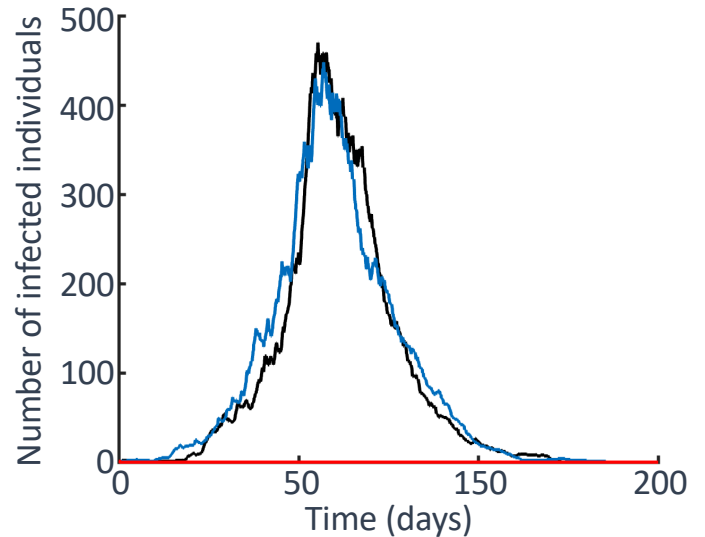
## Assessing the risk of a major epidemic



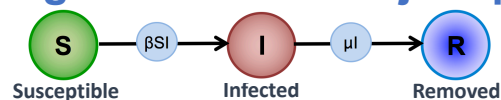
**Epidemic Risk:** the probability that an imported case leads to a major epidemic

If  $ER = 0$ ; a major epidemic will not occur

If  $ER = 1$ ; a major epidemic will definitely occur

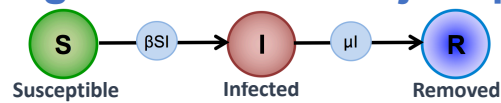


## Assessing the risk of a major epidemic



- Assume we start with one infected individual
- Denote  $q_i = \text{Prob}(\text{no major epidemic starting from } i \text{ infected individuals})$
- Want to find  $1 - q_1$

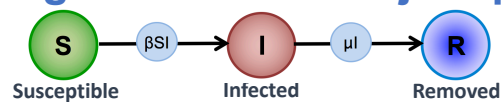
## Assessing the risk of a major epidemic



Two possibilities for the next event: infection or recovery

$$q_1 = \mathbb{P}(\text{infection}) \times q_2 + \mathbb{P}(\text{recovery}) \times q_0$$

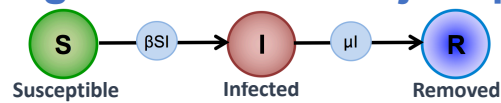
## Assessing the risk of a major epidemic



Two possibilities for the next event: infection or recovery

$$q_1 \approx \mathbb{P}(\text{infection}) \times q_1^2 + \mathbb{P}(\text{recovery})$$

## Assessing the risk of a major epidemic



Two possibilities for the next event: infection or recovery

$$q_1 \approx \mathbb{P}(\text{infection}) \times q_1^2 + \mathbb{P}(\text{recovery})$$

$$q_1 = \frac{1}{R_e} \text{ or } 1 \qquad ER = 1 - q_1 = 1 - \frac{1}{R_e}$$

### INTERFACE

royalsocietypublishing.org/journal/rsif

Research



Will an outbreak exceed available resources for control? Estimating the risk from invading pathogens using practical definitions of a severe epidemic

R. N. Thompson<sup>1,2</sup>, C. A. Gilligan<sup>3</sup> and N. J. Cunniffe<sup>3</sup>

PLOS COMPUTATIONAL BIOLOGY

RESEARCH ARTICLE

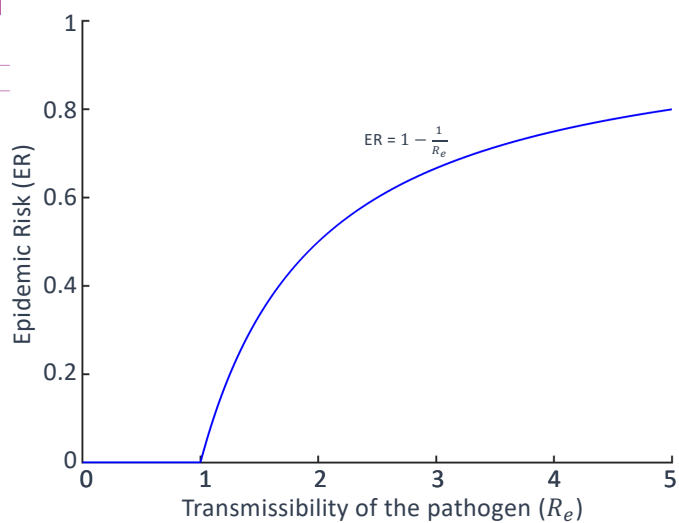
Detecting Presymptomatic Infection Is Necessary to Forecast Major Epidemics in the Earliest Stages of Infectious Disease Outbreaks

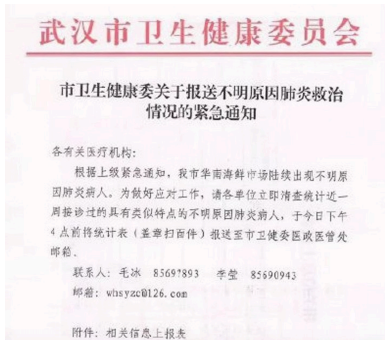
Robin N. Thompson<sup>\*</sup>, Christopher A. Gilligan, Nik J. Cunniffe

Sustained transmission of Ebola in new locations: more likely than previously thought

THE LANCET Infectious Diseases

Robin N Thompson, Katri Jalava, \*Uri Obolski





### How the virus has spread in China

■ No cases ■ 1 to 50 ■ 51 to 100 ■ 101 to 500 ■ More than 500

20 Jan: 291 cases



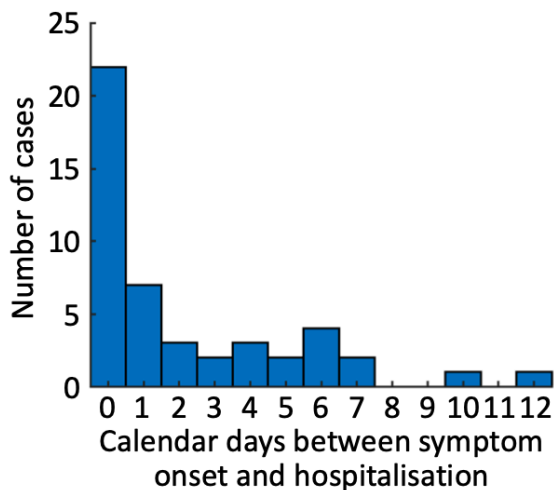
22 Jan: 446 cases



23 Jan: Line lists released (approx. 70 patients, incomplete data)

What is the epidemic risk outside of China?

$$R_e = \text{Infection rate} \times \text{Duration of infection}$$

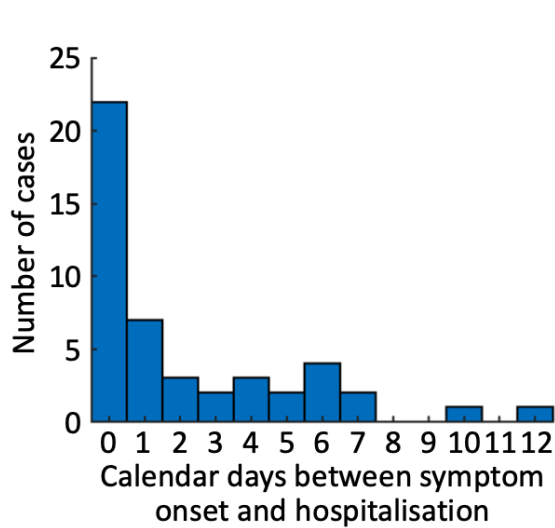


#### Interval-censored data

E.g. Symptoms – 10 Jan  
Hospitalisation – 11 Jan

Symptom onset to hospitalisation lies in the range  
0 – 2 days

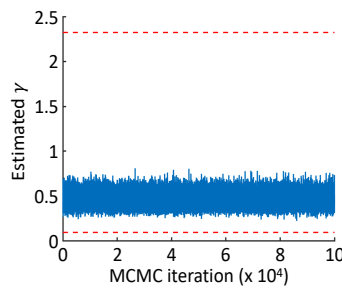
$$R_e = \text{Infection rate} \times \text{Duration of infection}$$



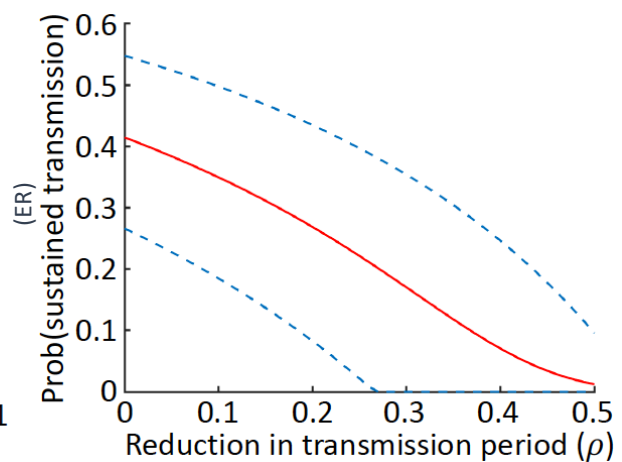
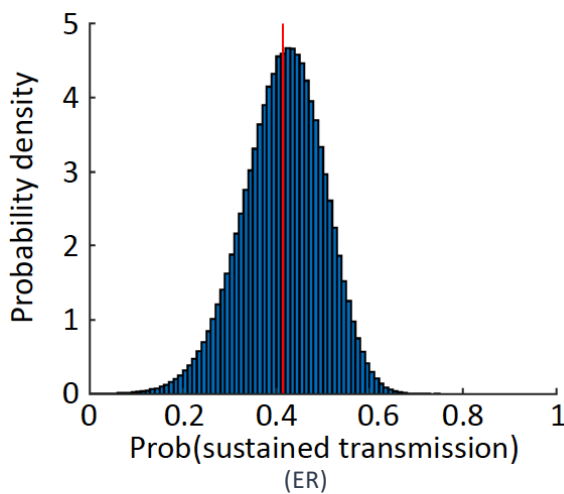
**Interval-censored data**

E.g. Symptoms – 10 Jan  
Hospitalisation – 11 Jan

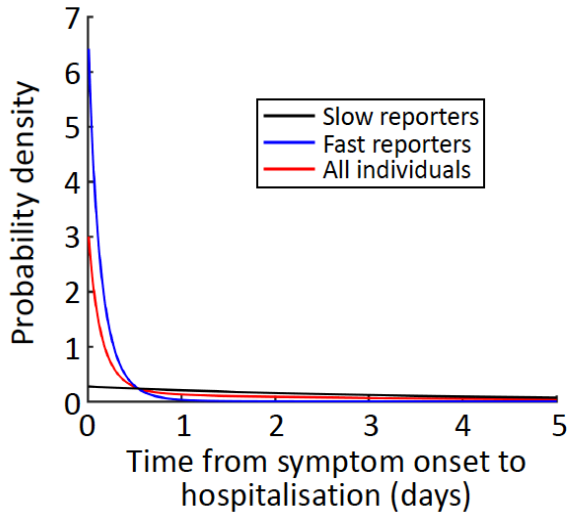
Symptom onset to hospitalisation lies in the range  
0 – 2 days



$$R_e = \text{Infection rate} \times \text{Duration of infection}$$



## Heterogeneity in reporting rates



## Heterogeneity in reporting rates

$q_{i,j}$  = Prob(no major epidemic |  $i$  fast reporters,  $j$  slow reporters)

$$q_{1,0} = \frac{\alpha\beta}{\beta + \gamma^{(1)}} q_{2,0} + \frac{(1-\alpha)\beta}{\beta + \gamma^{(1)}} q_{1,1} + \frac{\gamma^{(1)}}{\beta + \gamma^{(1)}} q_{0,0},$$

$$q_{0,1} = \frac{\alpha\beta}{\beta + \gamma^{(2)}} q_{1,1} + \frac{(1-\alpha)\beta}{\beta + \gamma^{(2)}} q_{0,2} + \frac{\gamma^{(2)}}{\beta + \gamma^{(2)}} q_{0,0}.$$

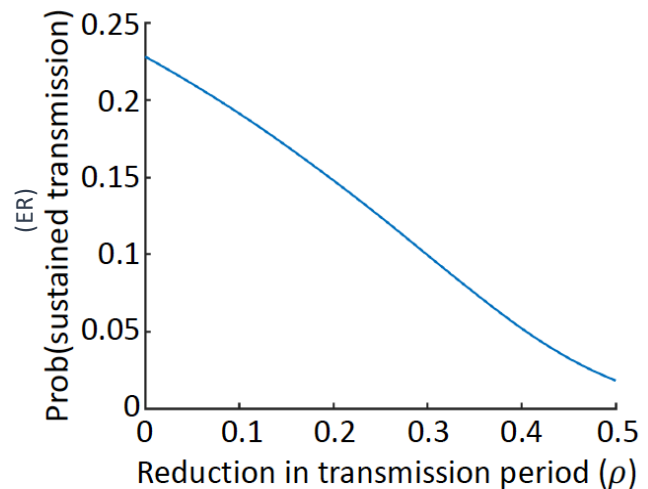
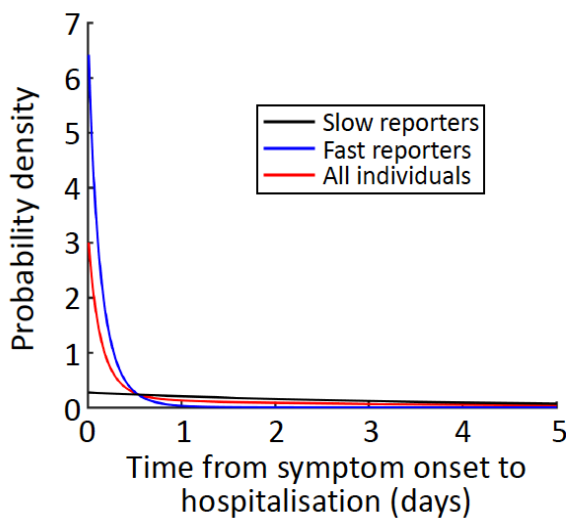
## Heterogeneity in reporting rates

$q_{i,j} = \text{Prob}(\text{no major epidemic} \mid i \text{ fast reporters}, j \text{ slow reporters})$

$$q_{1,0} = \frac{\alpha\beta}{\beta + \gamma^{(1)}} q_{1,0}^2 + \frac{(1-\alpha)\beta}{\beta + \gamma^{(1)}} q_{1,0}q_{0,1} + \frac{\gamma^{(1)}}{\beta + \gamma^{(1)}}$$

$$q_{0,1} = \frac{\alpha\beta}{\beta + \gamma^{(2)}} q_{1,0}q_{0,1} + \frac{(1-\alpha)\beta}{\beta + \gamma^{(2)}} q_{0,1}^2 + \frac{\gamma^{(2)}}{\beta + \gamma^{(2)}}$$

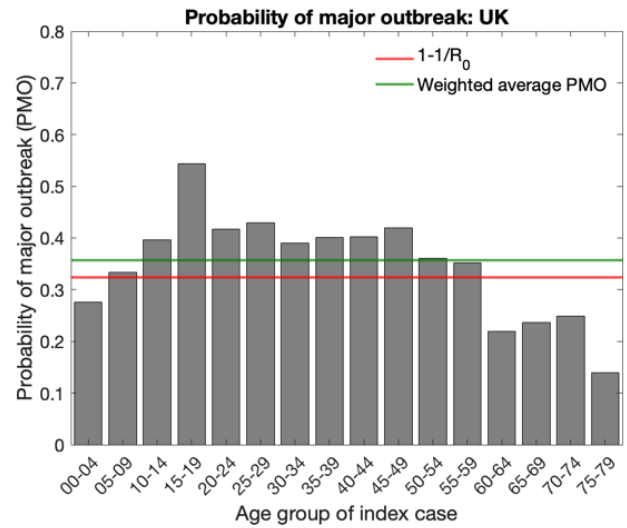
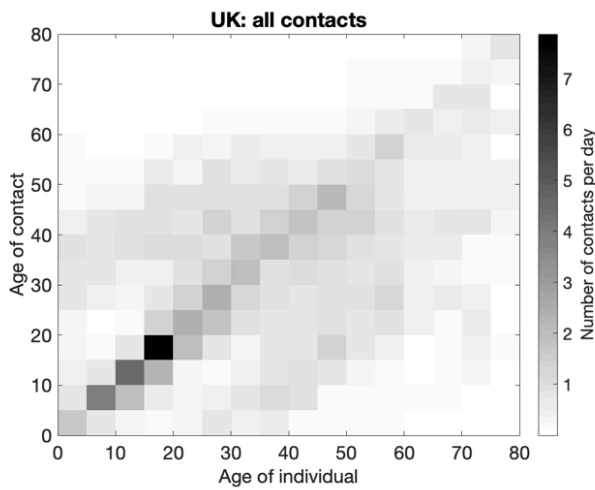
## Heterogeneity in reporting rates





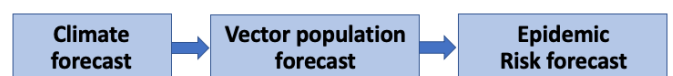
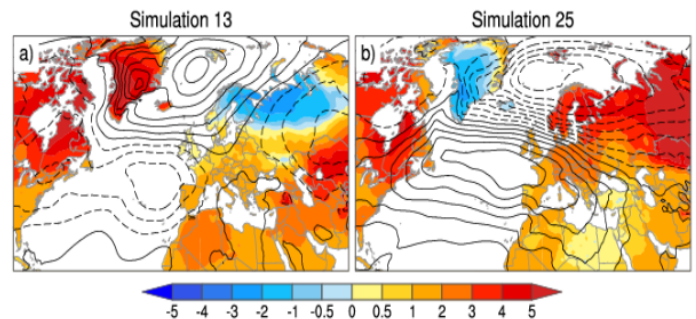
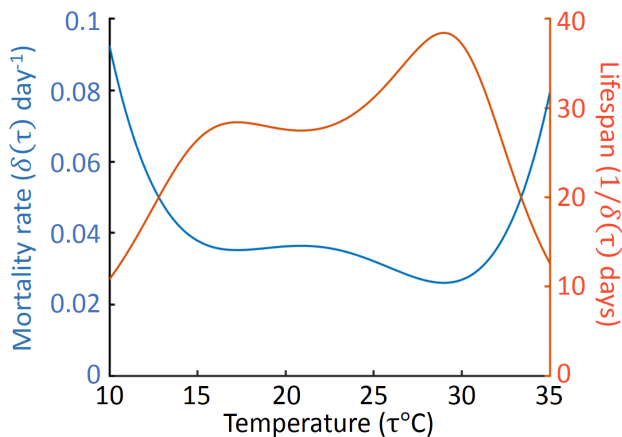
## Age structure

$$q_{i,j,k,\dots} = \text{Prob}(\text{no major epidemic} \mid i \text{ in age group 1, } j \text{ in age group 2, } k \text{ in age group 3, } \dots)$$



## Time-dependence

$$q(1, t) = q(2, t + \Delta t)\beta(t)N\Delta t + q(0, t + \Delta t)\mu\Delta t + q(1, t + \Delta t)(1 - \beta(t)N\Delta t - \mu\Delta t).$$



## Assessing Epidemic Risks – Summary

**Stochastic compartmental models** can be **used to estimate the Epidemic Risk** (the probability that an imported case leads to a major epidemic)

**Epidemic Risk estimates** can be **generated analytically**, informed by using outbreak data, and **adjusted in real-time**

**Estimates can be extended** to include a range of features, **including heterogeneity in reporting rates, age structure and temporal heterogeneity**

## Outline

### 1. Assessing the risk of a major epidemic

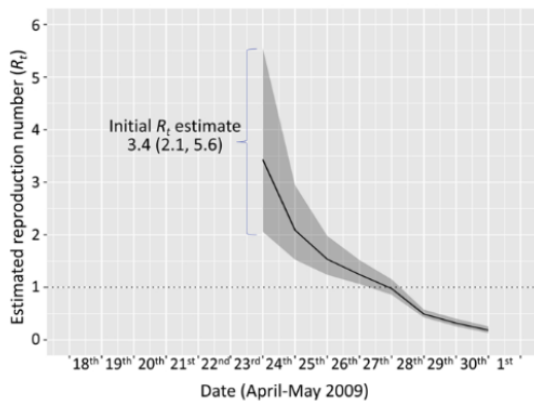
- Estimating the probability of a major epidemic
- Application to COVID-19
- More complex models: heterogeneity in reporting, age structure

### 2. Estimating changes in disease transmissibility

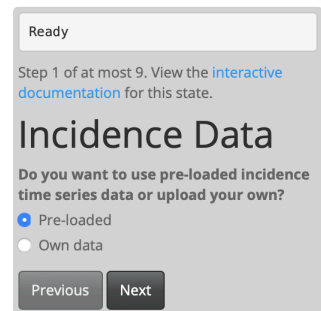
- Estimation of  $R_t$
- Extending the basic model
- Going forwards

**Middle**

- How effective are current interventions?
- Which interventions will minimise numbers of cases?



$$\begin{aligned}
 & \mathbf{P}(I_{t-\tau}^{\text{local}}, I_{t-\tau+1}^{\text{local}}, \dots, I_t^{\text{local}} \mid I_0, \dots, I_{t-\tau-1}, w_s, R_t) \\
 &= \prod_{k=t-\tau}^t \frac{(R_t \Lambda_k(w_s))^{I_k^{\text{local}}} \exp(-R_t \Lambda_k(w_s))}{I_k^{\text{local}}!} \\
 & \mathbf{P}(R_t \mid I_0, I_1, I_2, \dots, I_{t-\tau-1}, I_{t-\tau}^{\text{local}}, I_{t-\tau+1}^{\text{local}}, \dots, I_t^{\text{local}}, w_s) \\
 & \propto \mathbf{P}(I_{t-\tau}^{\text{local}}, I_{t-\tau+1}^{\text{local}}, \dots, I_t^{\text{local}} \mid I_0, \dots, I_{t-\tau-1}, w_s, R_t) \mathbf{P}(R_t)
 \end{aligned}$$

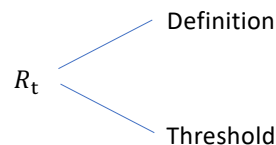


- Estimating changes in disease transmissibility

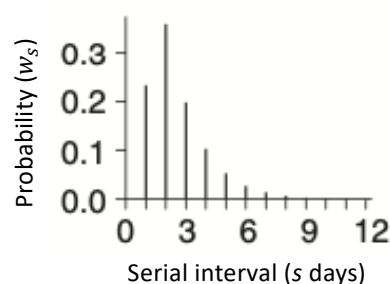
Thompson *et al.*, Epidemics, 2019

## Two important quantities

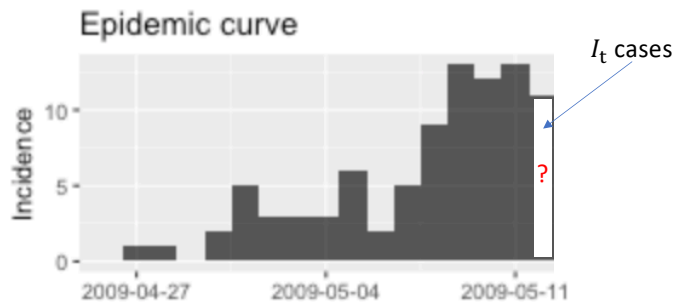
Time dependent reproduction number



Serial interval

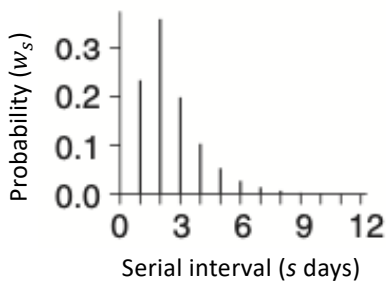


## Basic simulation model



Know

$R_t$  &

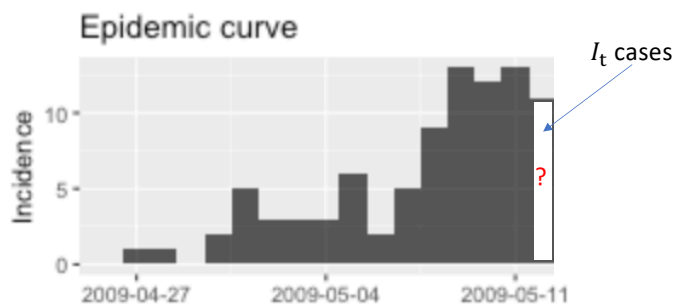


Basic model

$$E(I_t | R_t, \{w_s\}, \{I_0, I_1, I_2, \dots, I_{t-1}\}) = R_t \sum_{s=1}^t I_{t-s} w_s$$

$$P(I_t = x | R_t, \{w_s\}, \{I_0, I_1, I_2, \dots, I_{t-1}\}) = \frac{(E(I_t))^x \exp(-E(I_t))}{x!}$$

Cori et al., Am. J. Epi., 2013



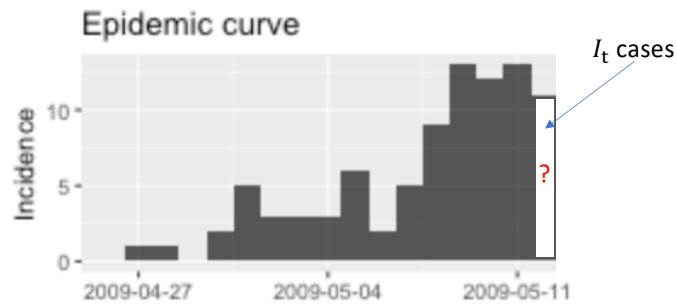
Bayes' rule:  $P(A | B) = \frac{P(B | A)P(A)}{P(B)}$

$P(I_t = x | R_t, \{w_s\}, \{I_0, I_1, I_2, \dots, I_{t-1}\})$



$P(R_t | I_t = x, \{w_s\}, \{I_0, I_1, I_2, \dots, I_{t-1}\})$

Cori et al., Am. J. Epi., 2013

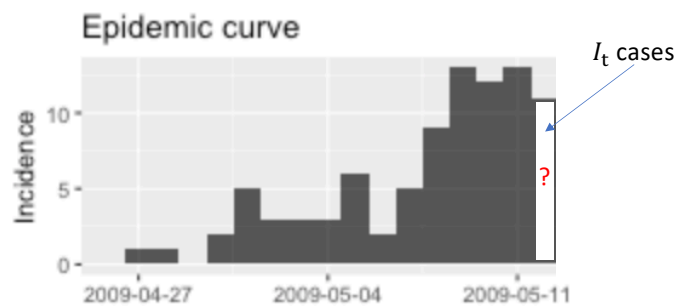


Bayes' rule:  $P(A | B) = \frac{P(B | A)P(A)}{P(B)}$

$P(I_t = x | R_t, \{w_s\}, \{I_0, I_1, I_2, \dots, I_{t-1}\})$        $\rightarrow$        $P(R_t | I_t = x, \{w_s\}, \{I_0, I_1, I_2, \dots, I_{t-1}\})$

Generates estimates of  $R_t$  that are highly sensitive to randomness in  $I_t$   
 Solution: Consider constant  $R_t$  over a window  $\{t - \tau, t - \tau + 1, \dots, t\}$

Cori et al., Am. J. Epi., 2013

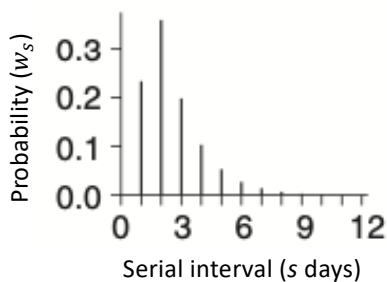
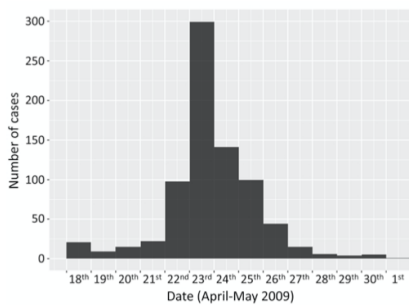


Bayes' rule:  $P(A | B) = \frac{P(B | A)P(A)}{P(B)}$

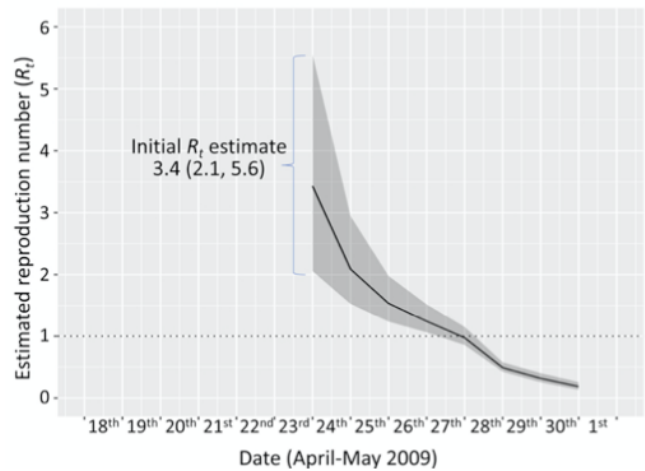
$P(I_{t-\tau} = x_{t-\tau}, \dots, I_t = x_t | R_t, \{w_s\}, \{I_0, I_1, I_2, \dots, I_{t-\tau-1}\})$        $\rightarrow$        $P(R_t | I_{t-\tau} = x_{t-\tau}, \dots, I_t = x_t, \{w_s\}, \{I_0, I_1, I_2, \dots, I_{t-\tau-1}\})$

Solution: Consider constant  $R_t$  over a window  $\{t - \tau, t - \tau + 1, \dots, t\}$

Cori et al., Am. J. Epi., 2013

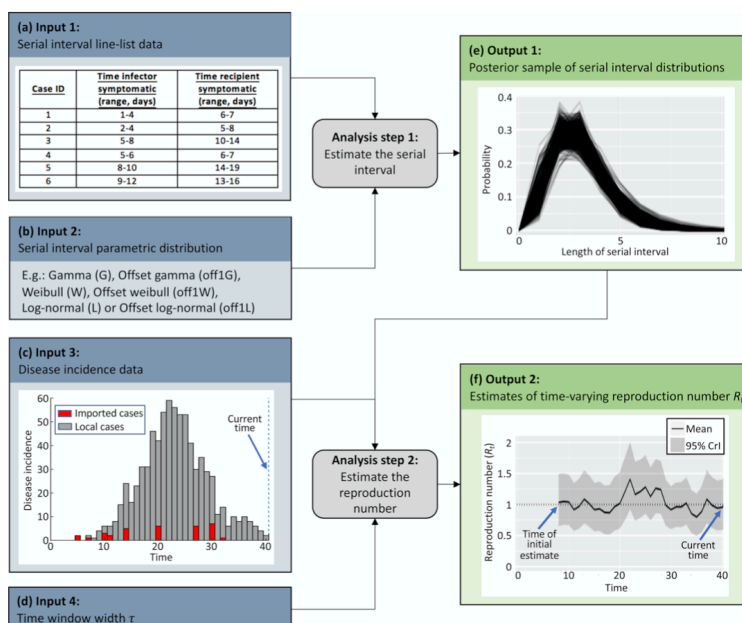


Window length:  $\tau$  days



Cori *et al.*, Am. J. Epi., 2013  
Thompson *et al.*, Epidemics, 2019

## Uncertainty in the serial interval, imported cases



Ready

Step 1 of at most 9. View the [Interactive documentation](#) for this state.

### Incidence Data

Do you want to use pre-loaded incidence time series data or upload your own?

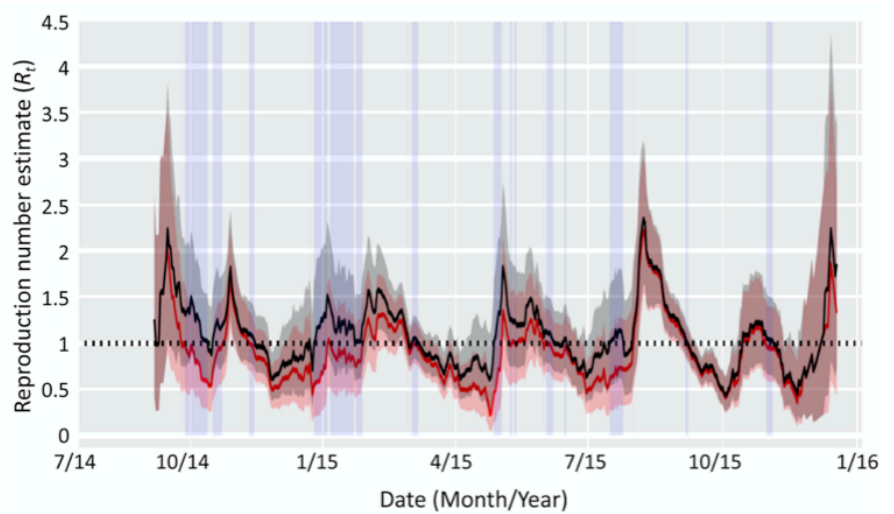
Pre-loaded  
 Own data

[Previous](#) [Next](#)

Thompson *et al.*, Epidemics, 2019

## Imported cases

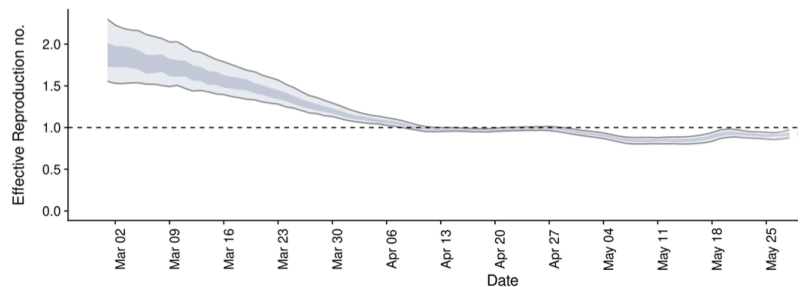
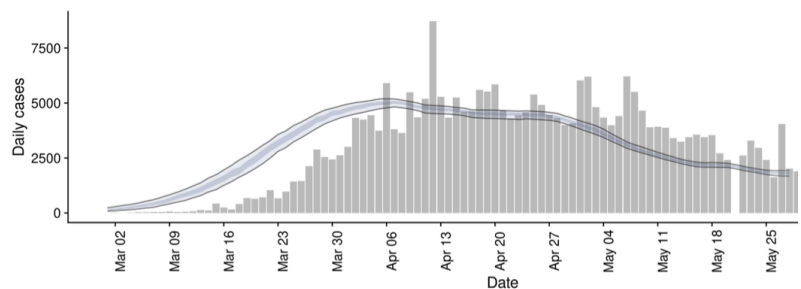
Imported cases have not been infected locally



Thompson *et al.*, Epidemics, 2019

## Conclusions – Estimating changes in transmissibility

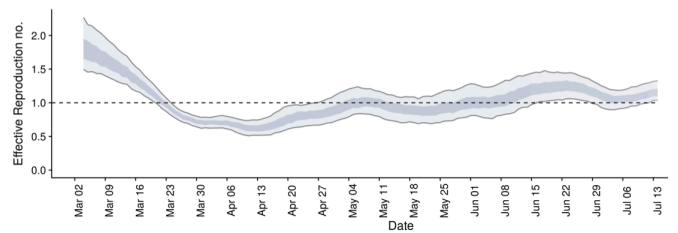
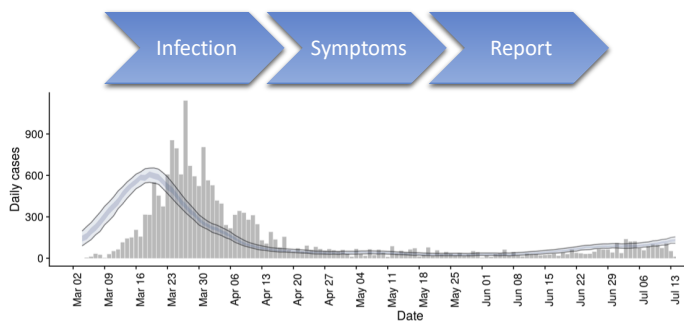
- Parameter inference can be used to estimate reproduction numbers in real-time during epidemics
- This approach has been used worldwide for COVID-19
- Population heterogeneity is important (e.g. local cases may have different characteristics compared to imported cases)





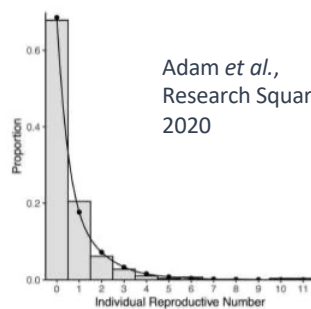
## Six challenges - Estimation of $R_t$

1. Unpicking effects of different measures
2. Reporting delays
3. Estimation when case numbers are low



PROCEEDINGS B Key questions for modelling COVID-19 exit strategies  
[royalsocietypublishing.org/journal/rspb](https://royalsocietypublishing.org/journal/rspb)  
Thompson *et al.*, Proc Roy Soc B, 2020

## Six challenges - Estimation of $R_t$

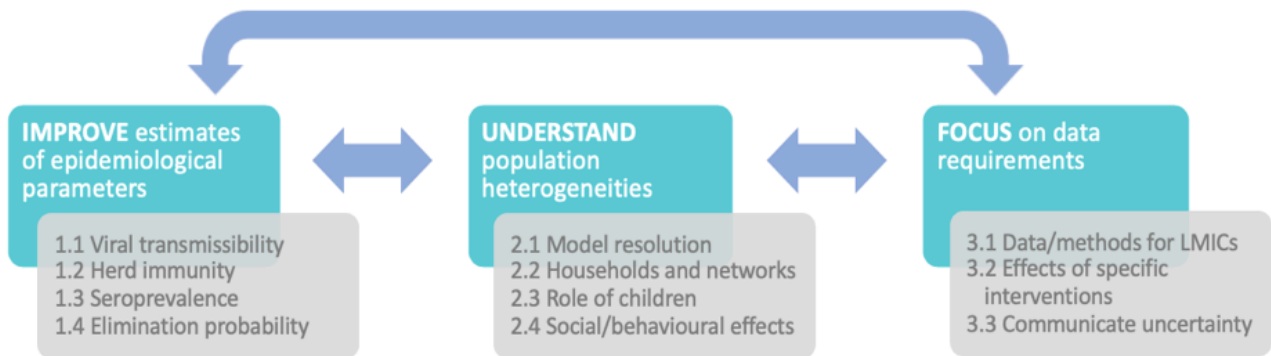


Adam *et al.*,  
Research Square,  
2020

4. Effect of heterogeneity within populations: i) different groups; ii) super-spreading
5. Asymptomatic transmission
6. Temporal changes in serial interval

PROCEEDINGS B Key questions for modelling COVID-19 exit strategies  
[royalsocietypublishing.org/journal/rspb](https://royalsocietypublishing.org/journal/rspb)  
Thompson *et al.*, Proc Roy Soc B, 2020

## Challenges going forwards



PROCEEDINGS B    Key questions for modelling COVID-19 exit strategies  
royalsocietypublishing.org/journal/rspb

Thompson *et al.*, Proc Roy Soc B, 2020

**Thanks!**