The Prominence of Stabilization Techniques in Column Generation: the case of Freight Transportation

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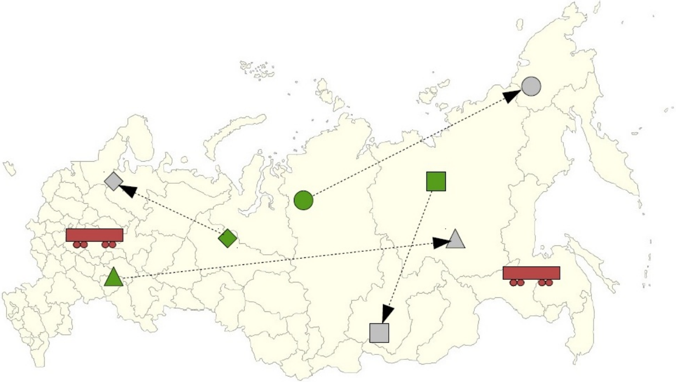
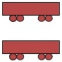
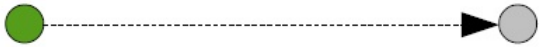
The freight car routing application: overview





initial car distribution



transportation demands

Specificity of freight rail transportation in Russia

The state company

- Freight car blocking
- Freight train scheduling
- Locomotives management
- Personnel management

Transp. costs matrix
Transp. times matrix

car movements

Independent freight car management companies

- Assignment of transportation demands to freight cars
- Freight car routing

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Transp. times matrix
→

←
car movements

Independent freight car management companies

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Distances are large, and average freight train speed is low ($\approx$ 300 km/day): discretization in periods of 1 day is reasonable

The freight car routing application: input and output

Input

- Railroad network (stations)
- Initial location of cars (sources)
- Transportation demands and associated profits
- Transportation times between stations
- Costs: transfer costs and standing (waiting) daily rates;

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Output: operational plan

- A set of accepted demands and their execution dates
- Empty and loaded cars movements to meet the demands (car routing)

Objective

Maximize the total net profit

Similar applications in the literature

[Fukasawa et al., 2002]

- Train schedule is known
- Cars should be assigned to trains to be transported
- Discretization by the moments of arrival and departure of trains.
- Smaller time horizon (7 days)

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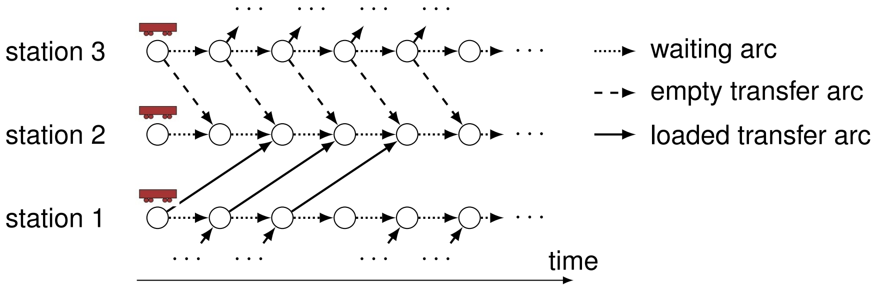
Other works

- [Holmberg et al., 1998]
- [Löbel, 1998]
- [Lulli et al., 2011]
- [Caprara et al., 2011]

Commodity graph [Stratonnikov and Shiryaev, 2012]

Commodity $c \in C$ represents the flow (movements) of cars of type c.

Graph $G_c = (V_c, A_c)$ for commodity $c \in C$:



Each vertex $v \in V_c$ represent location of cars of type c on a certain station at a certain time standing at a certain rate

g_a — cost of arc $a \in A_c$

Multi-commodity flow formulation

Notations

Q — set of demands,
C_q — set of car types compatible with demand $q \in Q$,
$n_q^{\max}$ — maximum number of cars to assign to demand $q \in Q$,
$\vec{n}_v^c$ — number of cars of type c situated initially in vertex $v \in V$.

Variables

$x_a^c \in \mathbb{Z}_+$ — flow size along arc $a \in A_c$, $c \in C$

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$$\min \sum_{c \in C} \sum_{a \in A_c} g_a x_a^c$$

$$\sum_{c \in C_q} \sum_{a \in A_{cq}} x_a^c \le n_q^{\max} \quad \forall q \in Q$$

$$\sum_{a \in \delta^-(v)} x_a^c - \sum_{a \in \delta^+(v)} x_a^c = \vec{n}_v^c \quad \forall c \in C, v \in V_c$$

$$x_a^c \in \mathbb{Z}_+ \quad \forall c \in C, a \in V_c$$

Contents

Path reformulation

- S_c — set of type c car "sources" $= \{v \in V_c : \vec{n}_v > 0\})$
- P_s^c — set of paths (type c car routes) from source $s \in S_c$

Variables

- λ_p — flow size along path $p \in P_s^c$, $s \in S_c$, $c \in C$

$$\min \sum_{c \in C} \sum_{s \in S_c} \sum_{p \in P_s} g_p^{path} \lambda_p$$

$$\sum_{c \in C_q} \sum_{s \in S_c} \sum_{p \in P_s^c :\ q \in Q_p^{path}} \lambda_p \le n_q^{\max} \quad \forall q \in Q$$

$$\sum_{p \in P_s^c} \lambda_p = \vec{n}_s^c \quad \forall c \in C, s \in S_c$$

$$\lambda_p \in \mathbb{Z}_+ \quad \forall c \in C, s \in S_c, p \in P_s^c$$

Column generation for path reformulation

- Pricing problem decomposes to shortest path problems, one for each source
  - slow: number of sources are thousands

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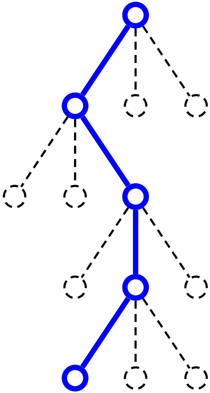
  - drawback: some demands are severely "overcovered", bad convergence
- We developed an iterative procedure:

repeat
  Find an in-tree T from all non-exhausted sources;
  foreach *path p in T in the order of its reduced cost* **do**
    Find n' — the maximum number of cars able to follow p;
    if $n' > 0$ **then**
      Add λ_p to the restricted master;
      Reduce by n' the number of cars in the source of p;
      Reduce by n' the volume of all demands covered by p;
until *iteration limit k* **or** *all demands are covered* **or** *all sources are exhausted*;

Diving Heuristic

Master problem solution λ^* can be fractional, so we apply the diving heuristic [Joncour et al., 2010]

- use Depth-First Search
- at each node of the tree
  - select *least fractional* column $\bar{\boldsymbol{\lambda}}_{\boldsymbol{p}}$: rounded value $\lceil \bar{\lambda}_p \rfloor > 0$
  - add $\lceil \bar{\lambda}_p \rfloor$ to the partial solution
  - update right-hand-side of the master constraints
  - apply preprocessing, which may lead to a change in the pricing problem variables bounds
  - solve the updated master LP



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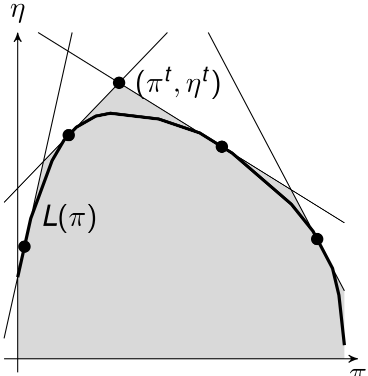
Column generation in the dual space

$L(\pi)$ — Lagrangian dual function



Outer approximation

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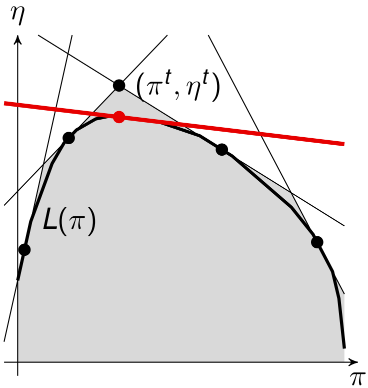

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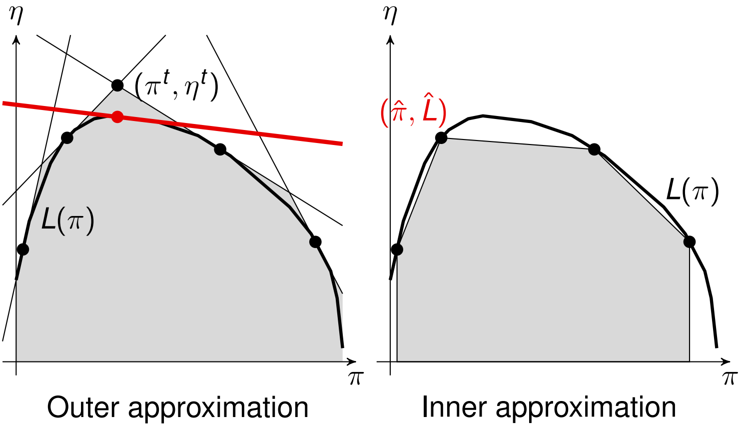


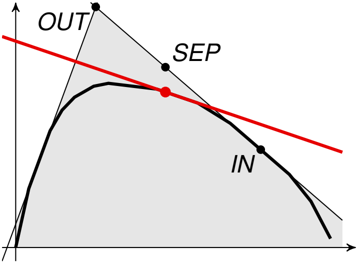
Dual Price Smoothing [Wentges, 1997]

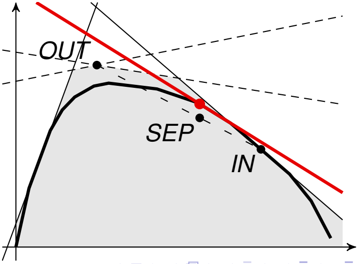
$$\boxed{\tilde{\pi}^t = \alpha\hat{\pi} + (1-\alpha)\pi^t}$$

$$(\pi^{\text{in}}, \eta^{\text{in}}) := (\hat{\pi}, \hat{L})$$

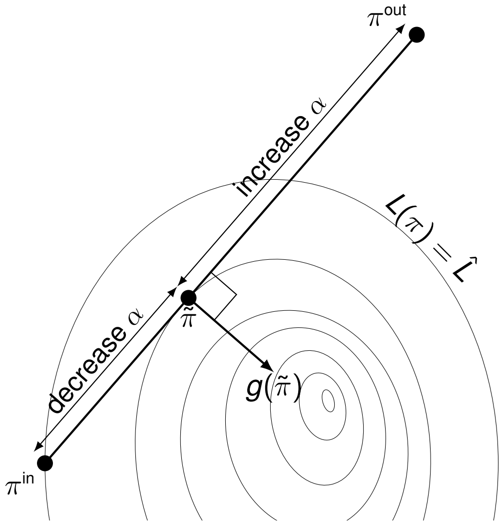
$$(\pi^{\text{out}}, \eta^{\text{out}}) := (\pi^t, \eta^t)$$

$$(\pi^{\text{sep}}, \eta^{\text{sep}}) := \alpha\,(\pi^{\text{in}}, \eta^{\text{in}}) + (1-\alpha)\,(\pi^{\text{out}}, \eta^{\text{out}})$$



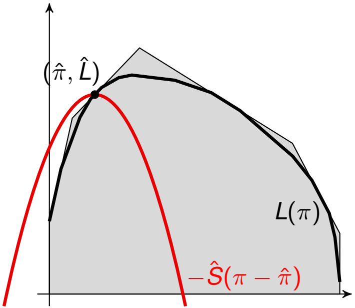


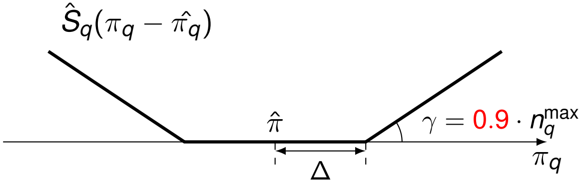
Auto-adaptative α-schedule [Pessoa et al., 2014]



Penalty functions [du Merle et al., 1999]

$$\boxed{\pi^t = \underset{\pi \in \mathbb{R}^m_+}{\operatorname{argmax}} \left\{ L^t(\pi) - \hat{S}_t(\pi) \right\}}$$





Here:

$$\Delta = \frac{\sum_{q \in Q} |\pi_q^1 - \hat{\pi}_q^0|}{|Q| \cdot \kappa}$$

Contents

Stabilization results

Real-life instances: 40-140 days horizon, 1,025 stations, up to 5,300 demands, 11 car types, 12,651 cars, and 8,232 sources. Up to $\approx$ 230 thousands nodes and 7.5 millions arcs.

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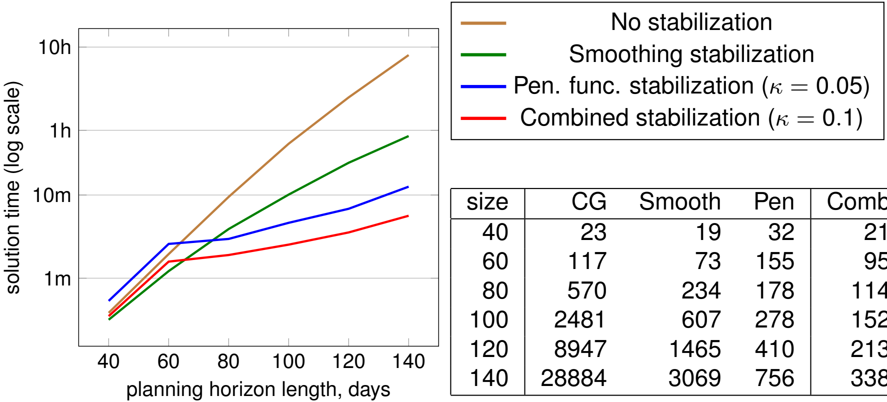

size	CG	Smooth	Pen	Comb
40	23	19	32	21
60	117	73	155	95
80	570	234	178	114
100	2481	607	278	152
120	8947	1465	410	213
140	28884	3069	756	338

Diving heuristic results

All instances solved to optimality by the diving heuristic.
(means Lagrangian bound is equal to the optimal solution value).

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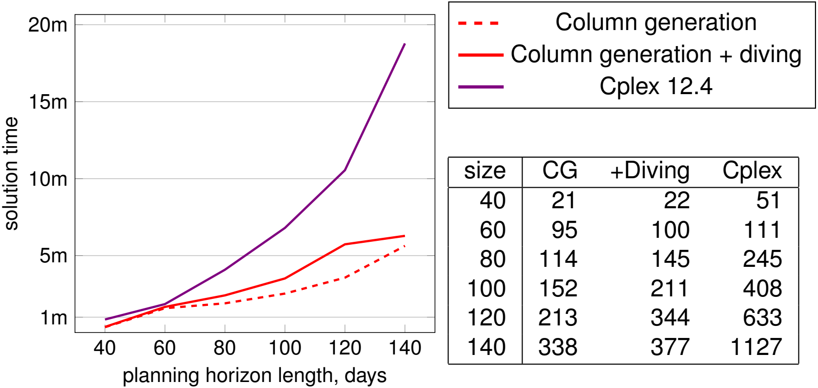

size	CG	+Diving	Cplex
40	21	22	51
60	95	100	111
80	114	145	245
100	152	211	408
120	213	344	633
140	338	377	1127

Conclusions

- A freight car routing application can be modelled as the multi-commodity flow problem
- Non-stabilized column generation implementation is not competitive with Cplex
- Generic combined stabilization with a single parameter gives up to 85x speed-up
- Generic diving heuristic allows us to obtain optimal integer solutions for real-life instances up to 3 times faster than Cplex.
- Column-and-row generation [Sadykov et al., 2013] gives better results that the stabilized column generation, but the diving heuristic cannot be directly applied

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