Stabilization in Column Generation: numerical study

Artur Pessoa[3] **Ruslan Sadykov**[1,2] Eduardo Uchoa[3]
François Vanderbeck[2,1]

[1] INRIA Bordeaux, France
[2] Univ. Bordeaux I, France
[3] Universidade Federal Fluminense, Brazil

ROADEF 2013
Troyes, France, February 13

Contents

Problem decomposition

Assume a bounded integer problem:

$$
\begin{array}{rrcl}
[\mathrm{F}] \equiv \min & c\,x & : & \\
 & A\,x & \geq & a \\
x \in Z = \{ & B\,x & \geq & b \\
 & x & \in & \mathbb{N}^n \quad \}
\end{array}
$$

Assume that subproblem

$$[\mathrm{SP}] \equiv \min\{c\,x : \; x \in Z\} \tag{1}$$

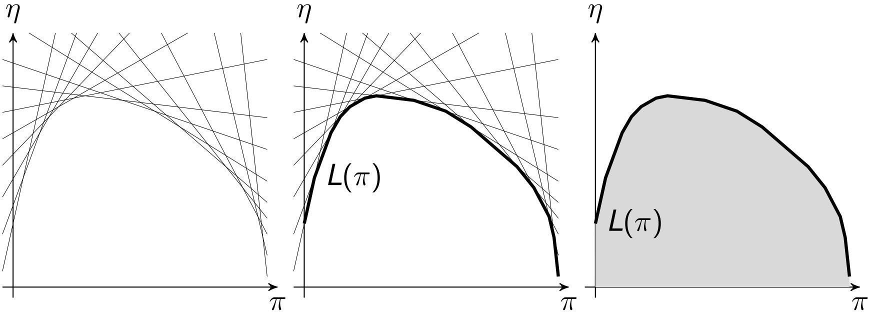
is <u>“relatively easy”</u> to solve compared to problem [F]. Then,

$$
\begin{array}{rcl}
Z & = & \{z^q\}_{q \in Q} \\
\mathrm{conv}(Z) & = & \{x \in \mathbb{R}^n_+ : \sum_{q \in Q} z^q \lambda_q, \; \sum_{q \in Q} \lambda_q = 1, \; \lambda_q \geq 0 \; q \in Q\}
\end{array}
$$

Lagrangian Relaxation & Duality

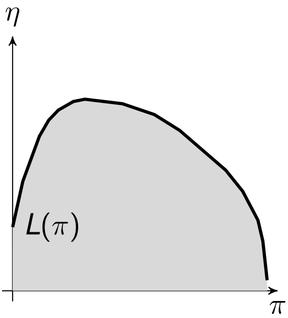
$$L(\pi) := \min_{q \in Q} \{c\, z^q + \pi\, (a - Az^q)\}$$

$$[\text{LD}] := \max_{\pi \in \mathbb{R}^m_+} \min_{q \in Q} \{c\, z^q + \pi(a - Az^q)\}$$



Lagrangian Dual as an LP

$$
\begin{aligned}
[\text{LD}] \quad & \equiv \quad \max_{\pi \in \mathbb{R}^m_+} \min_{q \in Q} \{\pi\, a + (c - \pi A) z^q\}; \\
& \equiv \quad \max\{\boldsymbol{\eta}, \\
& \qquad \boldsymbol{\eta} \le c z^q + \pi(a - A z^q) \quad q \in Q, \\
& \qquad \pi \in \mathbb{R}^m_+, \boldsymbol{\eta} \in \mathbb{R}^1\}; \\
& \equiv \quad \min\{\sum_{q \in Q} (c z^q) \lambda_q, \\
& \qquad \sum_{q \in Q} (A z^q) \lambda_q \ge a, \\
& \qquad \sum_{q \in Q} \lambda_q = 1, \\
& \qquad \lambda_q \ge 0 \qquad q \in Q\}; \\
& \equiv \quad \min\{cx : Ax \ge a,\ x \in \text{conv}(Z)\ \}.
\end{aligned}
$$



Dantzig-Wolfe Reformulation & Restricted Master

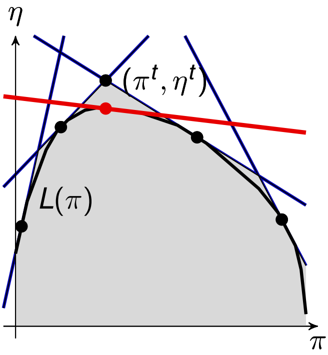
$$\begin{aligned} \min \sum_{q \in Q} (cx) \lambda_q & & \\ \sum_{q \in Q} (Az^q) \lambda_q & \geq & a \\ \sum_{q \in Q} \lambda_q & = & 1 \\ \lambda_q & \in & \{0,1\} \; \forall q \in Q \,. \end{aligned}$$

$$[\mathrm{M}^t] \equiv \min\{ \sum_{q \in Q^t} cz^q \lambda_q : \sum_{q \in Q^t} Az^q \lambda_q \geq a; \sum_{q \in Q^t} \lambda_q = 1; \lambda_q \geq 0, \; q \in Q^t \}$$

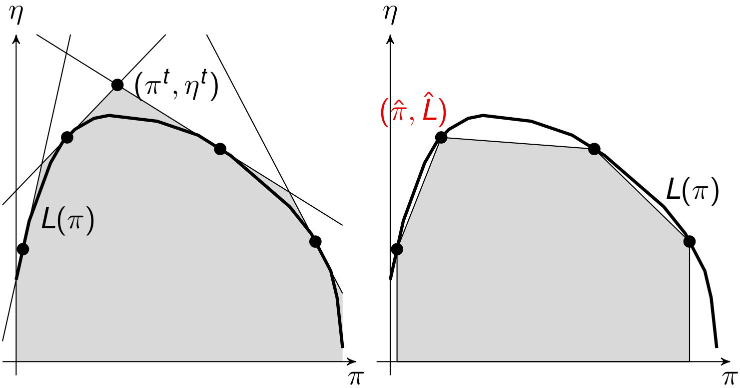
$$[\mathrm{DM}^t] \equiv \max\{ \eta : \pi(Az^q - a) + \eta \leq cz^q, \; q \in Q^t; \pi \in \mathbb{R}^m_+; \eta \in \mathbb{R}^1 \}$$

Restricted Master, Dual Polyhedra, & Pricing Oracle

- $[\mathrm{M}^t] \equiv \min \{cx : Ax \geq a, x \in \mathrm{conv}(\{z^q\}_{q \in Q^t})\}$.
- $L^t() : \pi \to L^t(\pi) = \min_{q \in Q^t} \{\pi a + (c - \pi A) z^q\}$;
- **Solving [LSP(π^t)]** yields:
  1. most neg. red. cost col. for $[\mathrm{M}^t]$
  2. most violated constr. for $[\mathrm{DM}^t]$
  3. a sub-gradient $g^t = (a - A\, z^t)$ of $L(.)$
  4. the correct value of $L(.)$ at point π^t



Dual Polyhedra: Outer and Inner approximations



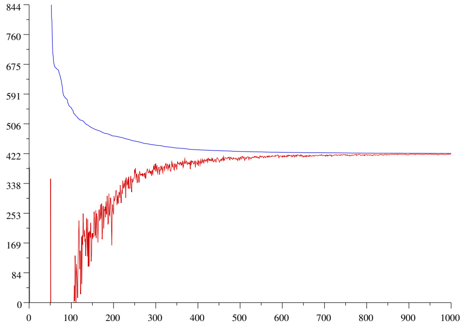
Convergence of Column Generation

- A sequence of candidate **dual solutions**

$$\{\pi^t\}_t \rightarrow \pi^*$$

- A sequence of candidate **primal solutions** (a by-product)

$$\{x^t\}_t \rightarrow x^*$$

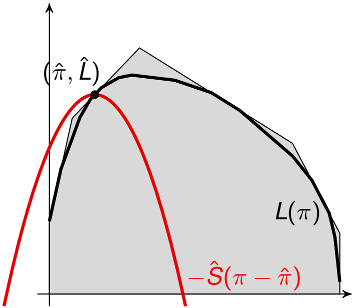


- **Dual oscillations**
- **Tailing-off effect**
- **Primal degeneracy**

Contents

Penalty functions

$$\boxed{\pi^t = \underset{\pi \in \mathbb{R}^m_+}{\text{argmax}} \left\{ L^t(\pi) - \hat{S}_t(\pi) \right\}}$$

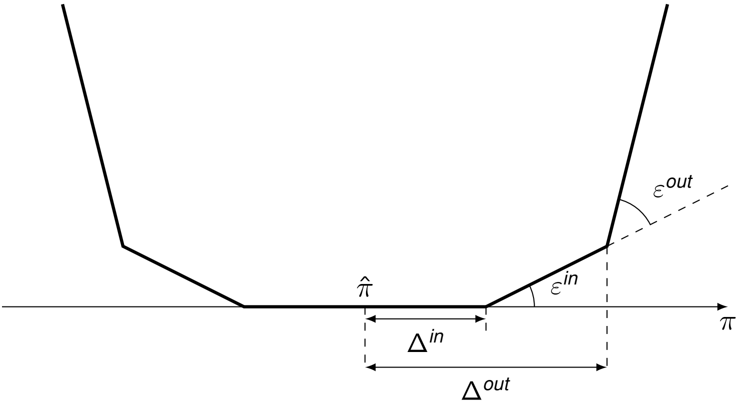


$$[\widehat{\mathrm{M}}^t] \quad \begin{array}{rcl} \min \sum_{q \in Q^t} c z^q \lambda_q + \hat{\pi}\,\rho & + & \hat{S}^*_t(\rho) \\ \sum_{q \in Q^t} A z^q \lambda_q + \rho & \geq & a \\ \sum_{q \in Q^t} \lambda_q & = & 1 \\ q & \in & Q^t \\ \lambda_q & \geq & 0 \end{array}$$

$$[\widehat{\mathrm{DM}}^t] \quad \begin{array}{rcl} \max \pi\, a + \eta & - & \hat{S}_t(\pi - \hat{\pi}) \\ \pi A z^q + \eta & \leq & c\, z^q \\ q & \in & Q^t \\ (\pi, \eta) & \in & \mathbb{R}^m_+ \times \mathbb{R}^1 \end{array}$$

Piecewise linear penalty functions

3-pieces: **[du Merle, Villeneuve, Desrosiers, Hansen 99]**
5-pieces: **[Ben Amor, Desrosiers, Frangioni 09]**



4 additional variables per master constraint.

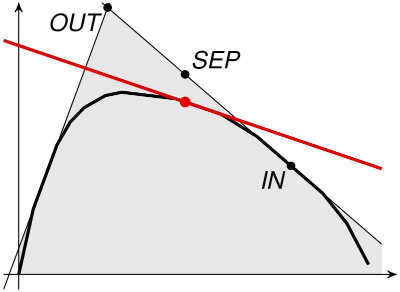
Dual Price Smoothing (I)

$$\boxed{\tilde{\pi}^t = \alpha\hat{\pi} + (1-\alpha)\pi^t}$$ **[Wentges 97]**

$$(\pi^{\text{in}}, \eta^{\text{in}}) := (\hat{\pi}, \hat{L})$$

$$(\pi^{\text{out}}, \eta^{\text{out}}) := (\pi^t, \eta^t)$$

$$(\pi^{\text{sep}}, \eta^{\text{sep}}) := \alpha\,(\pi^{\text{in}}, \eta^{\text{in}}) + (1-\alpha)\,(\pi^{\text{out}}, \eta^{\text{out}})$$

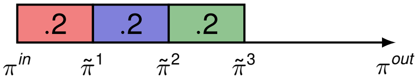


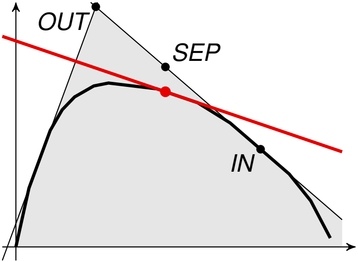
Dual Price Smoothing (II)

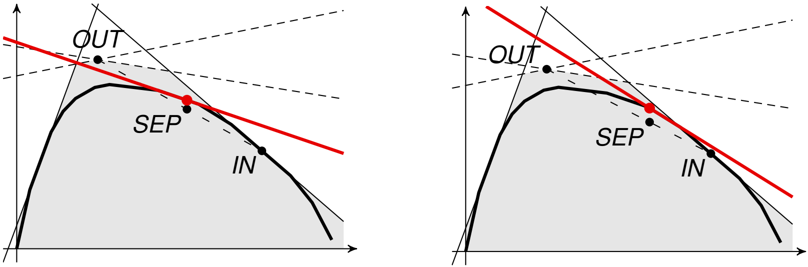
Case A: SEP is cut, so is OUT

Case B: SEP is not cut, but OUT is cut

Case C: neither SEP nor OUT is cut $\rightarrow$ "mis-price"





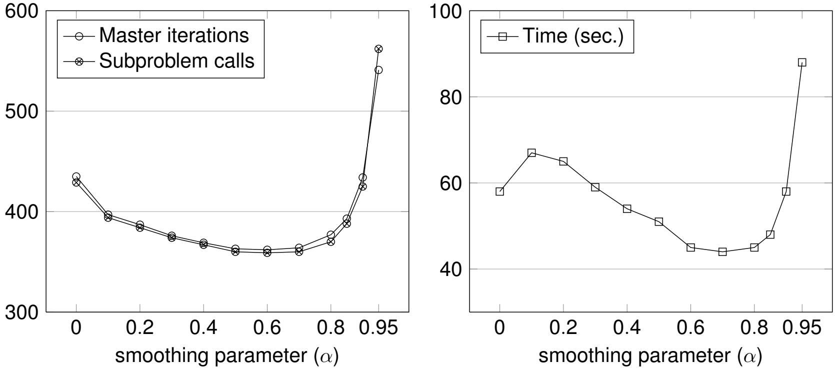


Smoothing with a static α (I)

Generalized assignment

OR-Library C, D, E instances: 10 jobs per agent

100 jobs / 10 agents; 200 jobs / 20 agents; 400 jobs / 40 agents

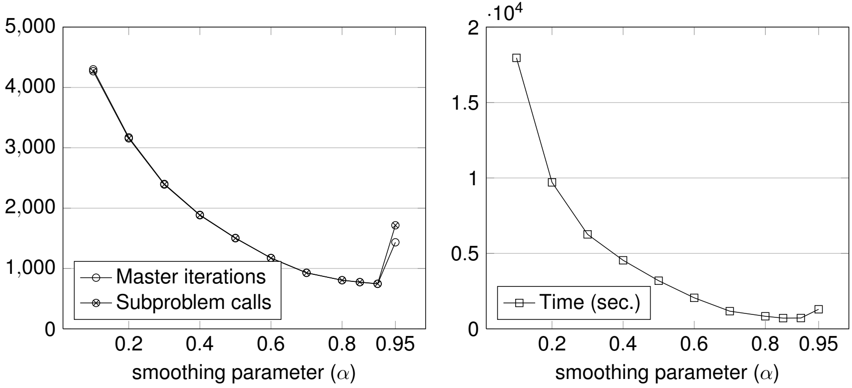


Smoothing with a static α (II)

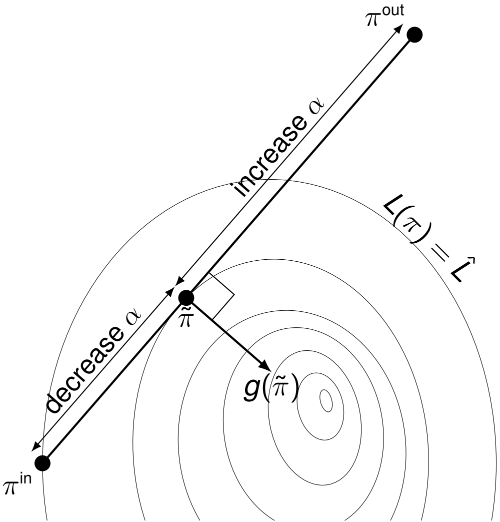
Generalized assignment

OR-Library C, D, E instances: 40 jobs per agent

200 jobs / 5 agents ; 400 jobs / 10 agents

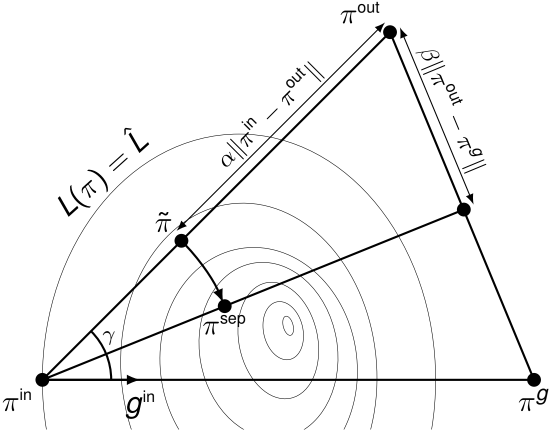


Smoothing: auto-adaptative α-schedule



Directional smoothing

hybridization with ascent methods



Automatic directional smoothing: $\beta = \cos\gamma$

Contents

Test problems

- **Parallel Machine Scheduling:** 30 instances generated in the same way as in the OR-Library with number of machines in $\{1, 2, 4\}$ and jobs in $\{50, 100, 200\}$.
- **Generalized Assignment:** 18 OR-Library instances of types D and E with number of agents in $\{5, 10, 20, 40\}$ and jobs in $\{100, 200, 400\}$.
- **Multi-Echelon Small-Bucket Lot-Sizing:** 17 randomly generated instances varying by the number of echelons in $\{1, 2, 3, 5\}$, items in $\{10, 20, 40\}$, and periods in $\{50, 100, 200, 400\}$.
- **Bin Packing:** 12 randomly generated instances with number of items in $\{400, 800\}$ and average number of items per bin in $\{2, 3, 4\}$.
- **Capacitated Vehicle Routing:** 21 widely used instances from the literature of types A, B, E, F, M, P with 50-200 clients and 4-16 vehicles.

Auto-adaptative Wentges Smoothing

- `It` — number of iterations in column generation
- `Col` — max. number of columns in master
- `Tim` — solution time

Geometric means are shown

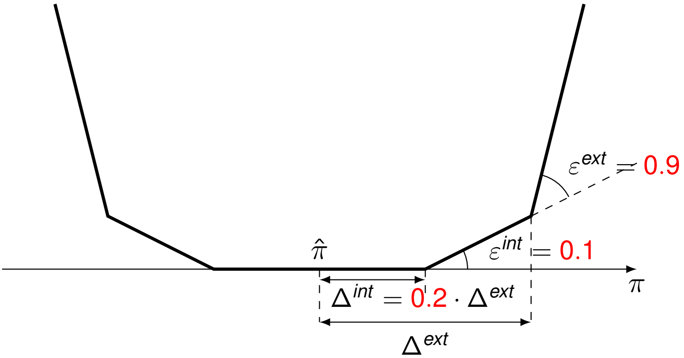
	Ratio $\alpha = 0$ vs $\alpha =$ *best*			Ratio $\alpha = 0$ vs $\alpha =$ *auto*		
Problem	`It`	`Col`	`Tim`	`It`	`Col`	`Tim`
Generalized Assignment	3.45	3.58	5.19	3.45	3.83	5.32
Lot-Sizing	2.16	2.84	3.08	2.40	3.56	4.26
Machine Scheduling	2.30	2.30	3.04	2.29	2.29	2.98
Bin Packing	1.54	1.48	1.79	1.49	1.45	1.65
Vehicle Routing	1.32	1.40	1.37	1.15	1.46	1.28

Directional Wentges Smoothing

	Ratio $\alpha = \textit{best}, \beta = 0$ vs $\alpha = \textit{best}, \beta = \textit{best}$			Ratio $\alpha = \textit{best}, \beta = 0$ vs $\alpha = \textit{auto}, \beta = \textit{auto}$		
Problem	`It`	`Col`	`Tim`	`It`	`Col`	`Tim`
General. Assignment	1.11	1.08	1.93	1.48	1.63	2.25
Lot-Sizing	1.17	1.27	1.50	1.37	1.69	1.83
Machine Scheduling	0.94	0.86	0.91	1.10	1.10	1.21
Bin Packing	0.95	0.95	0.94	1.04	1.03	0.96
Vehicle Routing	0.90	0.95	0.92	0.83	1.03	0.92

Penalty function setup

Hard to cover all parameter setting $\Rightarrow$ we concentrate on a good class of symmetric 5-piece linear functions:



- Δ^{int} is multiplied by κ each time the stability center changes
- ε^{int} is divided by 3 each time a artificial variable in the optimal solution
- Exhaustive search for best couple of parameters Δ^{ext}, κ (instance dependent)

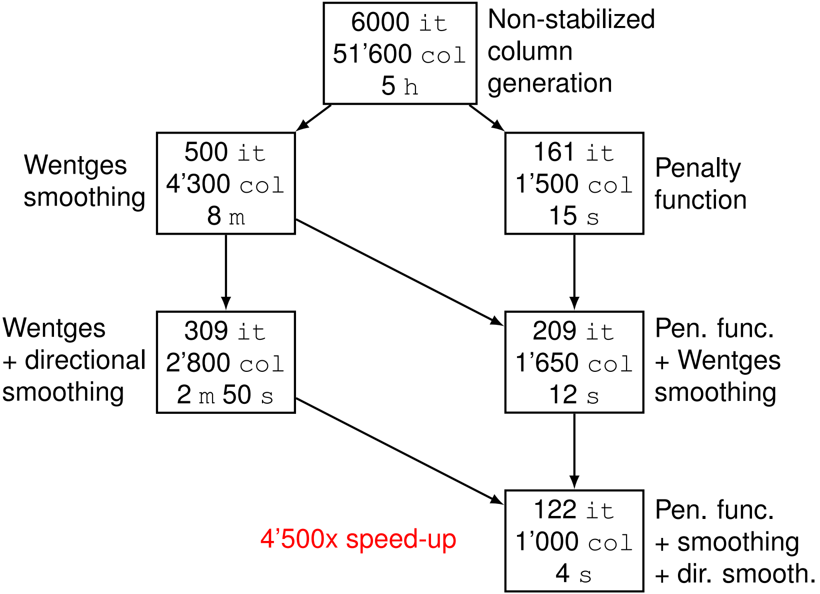
Smoothing vs. Penalty function stabilization

Are shown ratios of non-stabilized column generation versus *x*

Smoothing is with $\alpha = auto$, $\beta = auto$

x	It	Col	Tim	
Machine Scheduling				
Smoothing	2.53	2.52	3.68	$\leftarrow$ auto
Penalty function	2.19	2.19	3.47	$\leftarrow$ tuned
Penalty function + smoothing	3.08	3.08	6.05	$\leftarrow$ tuned
Generalized assignment				
Smoothing	5.00	5.70	10.0	$\leftarrow$ auto
Penalty function	8.10	8.62	31.1	$\leftarrow$ tuned
Penalty function + smoothing	7.67	9.94	53.4	$\leftarrow$ tuned

Instance GAP - D - 10agents - 400jobs



Instance GAP - D - 10agents - 400jobs