Beyond Hamilton-Jacobi, Last call to Bordeaux
Université de Bordeaux, Institut de Mathématiques
January 9 – 13, 2017

Scientific committee:

Marie-Claude Arnaud (Université d'Avigon, France)
Guy Barles (Université de Tours, France)
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Sponsors:

ANR-12-BS01-0020, "KAM Faible, au-delà de Hamilton-Jacobi"
CNRS, LIA France-Italie "Laboratoire Ypatie des Sciences Mathématiques"
Institut de Mathématiques de Bordeaux, IMB
ENS Lyon
Université d'Avigon
Institut Universitaire de France, IUF

Mini-courses

Andrea Davini
Università di Roma La Sapienza, Italy

Vincent Humilière
Université Pierre et Marie Curie, France
Valentine Roos
ENS Paris, France

Plenary lectures

Rodrigo Bissacot (University of São Paulo, Brasil)
Sergey Bolotin (University of Wisconsin, United States)
Piermarco Cannarsa (Università di Roma "Tor Vergata", Italy)
Wei Cheng (Nanjing University, China)
Xiaojun Cui (Nanjing University, China)
Jin Feng (The University of Kansas, United States)
Diogo Aguiar Gomes (KAUST University, Saudi Arabia)
Umberto Hryniewicz (Universidade Federal do Rio de Janeiro, Brasil)
Kei Irie (Kyoto University, Japan)
Hitoshi Ishii (Waseda University, Japan)
Vadim Kaloshin (University of Maryland , United States)
Konstantin Khanin (University of Toronto, Canada)
Olivier Ley (INSA Rennes, France)
Artur Lopes (Universidade Federal do Rio Grande do Sul, Brasil)
Ezequiel Maderna (Universidad de la República, Uruguay)
Hiroyoshi Mitake (Hiroshima University, Japan)
Yong-Geun Oh (IBS-CGP, Korea)
Antonio Siconolfi (Università di Roma "La Sapienza", Italy)
Claude Viterbo (ENS Paris, France)
Maxime Zavidovique (Université Pierre et Marie Curie, France)

Additional lectures

Alexandre Boritchev (Université de Lyon, France)
Eric Ossami Endo (Universidade de São Paulo, Brasil)
Renaud Leplaideur (Université de Bretagne Occidentale, France)
Stefan Suhr (Universität Hambourg, Germany)

Time table of the Conference

Monday 9

09:00 – 09:40	Hitoshi Ishii
09:50 – 10:30	Antonio Siconolfi
10:30 – 11:00	**break**
11:00 – 11:40	Diogo Gomes
11:50 – 12:30	Piermarco Cannarsa
12:30 – 14:00	**lunch**
14:00 – 14:50	V. Humilière, V. Roos
15:00 – 15:50	V.Humilière, V. Roos
16:00 – 16:30	**break**
16:30 – 17:10	Alexandre Boritchev
17:20 – 18:00	

Tuesday 10

09:00 – 09:40	Sergey Bolotin
09:50 – 10:30	Umberto Hryniewicz
10:30 – 11:00	**break**
11:00 – 11:40	Vadim Kaloshin
11:50 – 12:30	Claude Viterbo
12:30 – 14:00	**lunch**
14:00 – 14:50	V. Humilière, V. Roos
15:00 – 15:50	V. Humilière, V. Roos
16:00 – 16:30	**break**
16:30 – 17:10	Renaud Leplaideur
17:20 – 18:00	Stefan Suhr

Wednesday 11

09:00 – 09:40	Rodrigo Bissacot
09:50 – 10:30	Artur Oscar Lopes
10:30 – 11:00	**break**
11:00 – 11:40	Jin Feng
11:50 – 12:30	Konstantin Khanin
12:30 – 14:00	**lunch**
14:00 – 18:00	Social events

The dinner is at 20:00 and takes place at "La Belle Epoque", Quai Louis XVIII, 33000 Bordeaux, Tel. 05 56 79 14 58

Thursday 12

09:00 – 09:40	Xiaojun Cui
09:50 – 10:30	Wei Cheng
10:30 – 11:00	**break**
11:00 – 11:40	olivier Ley
11:50 – 12:30	Hiroyoshi Mitake
12:30 – 14:00	**lunch**
14:00 – 14:50	Andrea Davini
15:00 – 15:50	Andrea Davini
16:00 – 16:30	**break**
16:30 – 17:10	Eric Ossami Endo
17:20 – 18:00	

Friday 13

09:00 – 09:40	Kei Irie
09:50 – 10:30	Yong Geun Oh
10:30 – 11:00	**break**
11:00 – 11:40	Maxime Zavidovique
11:50 – 12:30	Ezequiel Maderna
12:30 – 14:00	**lunch**
14:00 – 14:50	Andrea Davini
15:00 – 15:50	Andrea Davini
16:00 – 16:30	**break**
16:30 – 17:10	
17:20 – 18:00	

Abstracts of the mini-courses

- Andrea Davini:

  Aubry Theory for systems of weakly coupled Hamilton-Jacobi equations

  Weakly coupled systems of Hamilton-Jacobi equations have been studied in the PDE literature as a particular instance of monotone systems, More recently, they have been considered in connection with asymptotic problems (homogenization, long-time behavior of solutions, etc.), where the analysis is based on the introduction of an associated critical value and on the study of the corresponding system, settled on a closed manifold. Such a critical value is unique, while the solutions of the critical system are not unique, even up to addition of a constant vector, in analogy with what happens for a single equation. In the latter case, the study of these nonuniqueness phenomena is part of weak KAM theory. In these lectures, we will present a weak KAM analogue for weakly coupled systems of Hamilton-Jacobi equations. In particular, we will introduce an adapted notion of Aubry set and we will present its main properties. The approach is purely based on PDE techniques.

- Vincent Humilière and Valentine Roos:

  Action selectors from symplectic topology and applications to Hamiltonian dynamics

  Actions selectors are maps that associate to any Hamiltonian function the action of one its trajectories, in a canonical and continuous way. They form a very convenient tool that can be used in many different situations in symplectic topology and Hamiltonian dynamics, in particular when one wants to drop the convexity assumption on the Hamiltonian. We will sketch the construction of action selectors, give their properties and present some of their applications, focusing on those presumably related to the interests of the audience. In particular, we plan to discuss symplectic capacities, symplectic homogenization, and with more details variational solutions to Hamilton-Jacobi equations and their links to viscosity solutions. An extended program will be posted later.

Abstracts of the Conference

- Rodrigo Bissacot, Universidade de São Paulo, USP, Brasil :

  Stability of the Phase Diagram on Gibbs/Equilibrium Theory

  We discuss the stability of the phase transition phenomenon on Gibbs/Equilibrium Theory. In particular, results for ferromagnetic Ising Models perturbed with spatially dependent external fields and also results for Equilibrium states on Countable Markov shifts.

- Alexnadre Boritchev, University of Lyon, France:

  Exponential convergence and hyperbolicity of the minimisers for random Lagrangian systems

  We consider the randomly forced Hamiton-Jacobi equation in the spirit of Khanin, Sinai and their collaborators from the Lagrangian point of view (long-time behaviour of the minimisers) as well as from the point of view of the statistical behaviour of the solutions (long-time convergence towards the stationary measure). In both cases there is a phenomenon of exponential convergence. A part of the presentation is about a joint work with K. Khanin (University of Toronto).

- Sergey Bolotin, Moscow Steklov Math. Inst., Moscow, and Univ. of Wisconsin-Madison, USA:

  Degenerate billiards in celestial mechanics

  In an ordinary billiard trajectories of a Hamiltonian system are elastically reflected when colliding with a hypersurface (scatterer). If the scatterer is a submanifold of codimension more than one, then collisions are rare. Trajectories with infinite number of collisions correspond to orbits of a lower dimensional discrete Lagrangian system. Degenerate billiards appear as limits of Hamiltonian systems of celestial mechanics in the limit of small masses. We prove the existence of solutions of such systems shadowing trajectories of degenerate billiards.

- Piermarco Cannarsa, University of Rome Tor Vergata, Italy:

  Generalized characteristics and propagation of singularities of solutions to Hamilton-Jacobi equations, Part I: topological properties

  Viscosity solutions of Hamilton-Jacobi-Bellman equations are nonsmooth functions which may fail to be differentiable on small sets. Such singularities, which play an important role for the underlying optimal control problem, have been analyzed from various viewpoints. Their dynamics can be described by generalized characteristics, which are forward solutions of the charactheristic system in Filippov's sense. In this talk, for stationary Tonelli Hamiltonians we develop an intrinsic proof of the existence of generalized characteristics using the positive Lax-Oleinik semigroup. This approach brings to light the topological structure of the singular set of a viscosity solution, which turns out to be locally path connected as shown in a recent joint work with W. Cheng and A. Fathi.

- Wei Cheng, Nanjing University, China :

  Generalized characteristics and singularitites of solutions to Hamilton-Jacobi equations, Part 2: Differential properties

  This talk is based on the joint work with Piermarco Cannarsa and Albert Fathi. It is also an extension to the talk by Piermarco. In this part, we will talk on the propagation of singularities along the generalized characteristics di erential inclusions. In particular, we will discuss the asymptotic behavior of the singularities along the generalized characteristics and its connection to the regular dynamics.

- Xiaojun Cui, Nanjing University, China :

  Viscosity solutions of eikonal equations

  For a noncompact complete Riemannian manifold, we relate some topological and geometric notions to the structure of viscosity solutions of the eikonal equation. Viscosity solutions of eikonal equations for some spacetimes are also considered.

- Eric Ossami Endo, University of Sao Paulo, Brazil - University of Groningen, Netherlands :

  Phase Transitions: the case of Inhomogeneous External Fields Ising model on Cayley Trees

  Ferromagnetic Ising model has been studied since the seminal paper of E. Ising, which he proved the absence of the phase transition of the model on the lattice $\mathbb{Z}$. R. Peierls proved the existence of phase transition on the lattice $\mathbb{Z}^d$ when $d > 1$, contradicting the conjecture of Ising, who conjectured that the Ising model has no phase transition for any $d \geqslant 1$. Adding an external field on the Ising model, T. D. Lee and C. N. Yang proved the absence of phase transition on Ising model for any $d > 1$ when the external field is constant and non-zero. The papers of R. Bissacot, M. Cassandro, L. Cioletti and E. Presutti, together with the work of L. Cioletti and R. Vila, showed conditions to the spatially dependent external fields decaying to zero in order to the ferromagnetic Ising model undergoes the phase transition. On Cayley Trees in opposed to the situation on $\mathbb{Z}^d$, the phase transition on Cayley trees can appear even when the model has a non-zero homogeneous external field, this concludes that there exists the critical external fields $h_c > 0$ separating the presence and the absence of the phase transition. As the same question in the work of R. Bissacot, M. Cassandro, L. Cioletti and E. Presutti, we are going to show the condition to separate the presence and the absence of the phase transition on the spatially dependent external fields Ising model on Cayley trees when the external fields are decaying to the critical external field h_c. We use in the proof some dynamical systems tools, such as the stability of the plus state, which is a saddle node on the critical external field, under addition of a negative spatially dependent perturbation on the external fields.

- Jin Feng, University of Kansas, USA:

  A metric space analysis approach to a Hamilton-Jacobi equation in the space of probability measures

  Hamilton-Jacobi equations in the space of probability measure is a class of very singular PDEs. We start this talk with two examples. The first one concerns a statistical mechanics application where the equation is derived by Dutch probabilists studying Gibbs-Non-Gibbs transitions. Through a method developed by the speaker and Tom Kurtz, the equation's uniqueness implies a rigorous probabilistic large deviation principle. However, this uniqueness problem is open. This model equation has a hidden spatial translation invariance. The second example concerns a variational formulation of a compressible Euler equation in the continuum mechanics. The probability measure here represents density profile of infinite particles. A particle permutation invariance is present in the model. For this second example, we have a recently developed well-posedness theory.

  In the rigorous part of this talk, I focus on explaining how the well-posedness in the second example is solved. We view the space of probability measure as an infinite dimensional quotient space and use techniques based on metric analysis and the Wasserstein spaces. A key step is the use of a geometric tangent cone concept. Admissible velocity space for probability-measure valued curves is identified as the space of Markov transition kernels. This allows us to distinguish different curves with mass condensation property. A corresponding viscosity solution theory explores an identity valid in the geodesic metric spaces.

  This talk is based on a joint work with Luigi Ambrosio.

- Diogo Gomes, KAUST Saudi Arabia:

  Variational inequalities and mean-field games

  We consider stationary monotone mean-field games (MFGs) and study the existence of weak solutions. We introduce a regularized problem that preserves the monotonicity and prove the existence of solutions to the regularized problem. Next, using Minty's method, we establish the existence of solutions for the original MFGs. Finally, we examine the properties of these weak solutions in several examples.

- Umberto Leone Hryniewicz, UFRJ, Brasil and Ruhr-Universität Bochum, Germany:

  Linking and global surfaces of section

  In this talk we explain how Schwarzman-Fried-Sullivan's theory and Hofer-Wysocki-Zehnder's theory of pseudo-holomorphic curves in symplectizations meet to attack the following question: When does a collection of periodic orbits of a non-vanishing vector field (on a 3-manifold) bound a global surface of section? We provide a complete answer to this question for Reeb flows in planar contact 3-manifolds, extending a classical result of Birkhoff for positively curved metrics on the two-sphere. This is joint work with Pedro Salomao and Kris Wysocki.

- Kei Irie, RIMS, Kyoto Unviersity, Japan:

  A C^∞ closing lemma for three-dimensional Reeb flows via embedded contact homology

  Embedded contact homology (ECH) is a version of Floer homology defined for contact three-manifolds. I will explain an application of this theory to Reeb dynamics; a proof of a C^∞ closing lemma for three-dimensional Reeb flows.

  The key ingredient of the proof is a result by Cristofaro-Gardiner, Hutchings and Ramos, which claims that the asymptotics of ECH spectral invariants recover the volume of a contact manifold.

  Applications to closed geodesics on Riemannian two-manifolds and Hamiltonian diffeomorphisms of symplectic two-manifolds are also presented. If time permits, I will also discuss some possible extensions and questions.

- Hitoshi Ishii, Waseda University, Japon:

  The vanishing discount problem for fully nonlinear degenerate elliptic PDEs

  I explain an approach, based on generalized Mather measures, to the vanishing discount problem for fully nonlinear, degenerate elliptic, partial differential equations. Under mild assumptions, we introduce viscosity Mather measures for such PDEs, which are natural extensions of Mather measures, originally due to J. Mather. Using the viscosity Mather measures, one can show that the whole family of solutions v^λ of the discount problem, with the discount factor λ, converges to a solution of the ergodic problem as λ goes to 0. An emphasis of my talk will be on the Neumann boundary value problems. The talk is based on joint work with Hiroyoshi Mitake (Hiroshima University) and Hung V. Tran (University of Wisconsin-Madison).

- Vadim Kaloshin, University of Maryland, USA and ITS-ETH, Switzerland:

  On local Birkhoff conjecture for convex billiards

  The classical Birkhoff conjecture states that the only integrable billiard is the billiard inside an ellipse. We show that this conjecture is true for small perturbations of ellipses preserving many rational caustics. This consists of two main steps: study small perturbations of the circle (joint with A. Avila and J. De Simoi) and extend the analysis to small perturbation of ellipses (joint with A. Sorrentino). In a somewhat different direction we prove the conjecture that for small perturbation of the circle preserving not so many rational caustics (joint w G. Huang and A. Sorrentino).

- Konstantin Khanin, University of Toronto, Canada:

  On minimizers or random Lagrangian systems

  In this talk I'll review results on global solutions to the random Hamilton-Jacobi equation in compact setting, and then discuss the problems and conjectures in the non-compact case. This is an area where the dynamical and probabilistic methods interact in an interesting and meaningful way. I'll also discuss a connection to the problem of KPZ universality.

- Renaud Leplaideur, University of Bretagne Occidentale, France:

  Easy Ising

  We give a kind of dictionary between the thermodynamic formalism for the Curie-Weiss model (mean field) in Probability and Statistical Mechanics on the one hand and Ergodic Theory on the other hand.

  More precisely, we show that for the Curie-Weiss model (mean field) a Gibbs measure associated to an Hamilton of finite range converges to a convex combination of (dynamical) conformal measures associated to the potential which generates the Hamiltonian.

  More suprising, we introduce a new concept of pressure (the quadratic pressure) and explain why ergodicists should maximize this quantity instead of the usal linear pressure.

- Olivier Ley, IRMAR-INSA of Rennes, France:

  Remarks for the large time behavior of solutions of Hamilton-Jacobi equations in the unbounded case

  I will discuss some results about the large time behavior for unbounded solutions of first-order Hamilton-Jacobi in the whole space R^n and illustrate the additional difficulties with respect to the periodic case on an explicit easy control problem. This is a joint work with Thi Tuyen Nguyen (Rennes) and Thanh Viet Phan (HoChiMinh City).

- Artur Oscar Lopes, Instituto de Matematica UFRGS, Brasil:

  Ergodic Transport and Ergodic Optimization

  We will present several new results on Transport Theory which are related to dynamical invariance, minimizing probabilities and Gibbs states.

- Ezequiel Maderna, Universidad de la Republica, Montevideo (Uruguay):

  Applications of weak KAM theory to some gravitational problems

  The weak KAM theory has been applied to the classicaln body problem. The main result obtained on the dynamics was the abundance of motions of the completely parabolic type. Although this is in itself interesting, we believe that the theory should give more results on the description of the dynamics of this classical problem as well as for other gravitational problems, and more particularly about their integrability or not in the classical sense (Liouville). The purpose of this talk will be to describe the state of the art in this research line.

- Hiroyoshi Mitake, Hiroshima university, Japan:

  Weak KAM theory for discount Hamilton-Jacobi equations and its application

  This is a joint work with Kohei Soga (Keio university). Weak KAM theory for discount Hamilton-Jacobi equations and corresponding discount Lagrangian/Hamiltonian dynamics is developed. Then it is applied to error estimates for viscosity solutions in the vanishing discount process. The main feature is to introduce and investigate the family of α-limit points of minimizing curves, with some details in terms of minimizing measures. In error estimates, the family of α-limit points is effectively exploited with properties of the corresponding dynamical systems.

- Yong-Geun Oh, IBS Center for Geometry and Physics & POSTECH, Korea:

  Topological extension of Calabi invariants and Lagrangian Floer theory

  In this talk, I will explain how one could employ the analytic machinery of Lagrangian Floer homology in study of continous Hamiltonian dynamics, expecially towards a topological extension of Calabi invariants and the application to the simpleness question of the area-preserving homeomorphism group of the 2-disc.

- Antonio Siconolfi, Università di Roma La Sapienza, Italy:

  Hamilton–Jacobi equations on networks

  We propose a new methodology to study Hamilton–Jacobi equations on networks immersed in an Euclidean space or possibly a manifold. The key feature is to combine it with a discrete functional equation on the vertices of an underlying finite graph.

  The discrete equation can be advantageously handled through simple combinatorial techniques, and the outputs are thereafter transferred to the continuous problem. This allows obtaining comparison results without using the usual double variable method, to write down representation formulae, and establish regularity properties.

  We apply this approach to Eikonal and discounted equations, the latter also in the nonconvex case. We formulate an adapted version of Weak KAM theory, and study the asymptotic behavior of the solutions to the discounted problem as the discount factor goes to 0.

  Research in collaboration with Alfonso Sorrentino.

- Stefan Suhr, Universität Hambourg, Germany:

  Optimal transportation for Lorentzian manifolds

  This talk will give a short introduction to optimal transportation in the context of Lorentzian geometry. Results include existence of minimal couplings, a sharp criterion for the existence of finite couplings, an intermediate regularity theorem and a solution to the Monge problem.

- Claude Viterbo, ENS Paris, Université Paris-Sud, Orsay, France:

  Barannikov-Morse complex, bar-codes and applications to Witten-Laplacian We explain how a Morse function gives rise to a canonical complex that is chain equivalent to the Morse complex. This complex determines the asymptotics of eigenvvalues for the Witten Laplacain, but has also a number of other applications.

- Maxime Zavidovique, IMJ-PRG (Paris 6), France:

  Dynamical aspects of weekly coupled systems of Hamilton-Jacobi equations We will present representation formulas for solutions of degenerate weakly coupled systems of Hamilton-Jacobi equations. They generalize the Lax-Oleinik formula for a single Hamilton-Jacobi equation. We will also review some results on how those formulas allow to give dynamical characterizations of Aubry sets. The first part is based on results obtained with Andrea Davini and Antonio Siconolfi.

List of participants

Denise Aregba	University of Bordeaux, France
Marie-Claude Arnaud	Iniversity of Avigon, France
Guy Barles	University Francois Rabelais Tours, France
Krisztian Benyo	University of Bordeaux, France
Patrick Bernard	University Paris Dauphine et ENS, France
Rodrigo Bissacot	Universidade Sao Paulo, USP, Brasil
Sergey Bolotin	University of Wisconsin-Madison, USA
Alexandre Boritchev	University of Lyon, France
Piermarco Cannarsa	University of Rome Tor Vergata, Italy
Wei Cheng	Nanjing University, China
Xiaojun Cui	Nanjing University, China
Andrea Davini	Universita di Roma La Sapienza, Italy
Eric Ossami Endo	Universidade Sao Paulo, Brasil
Albert Fathi	ENS Lyon, France
Jacques Féjoz	University Paris-Dauphine, Observatoire of Paris, France
Jin Feng	University of Kansas, USA
Anna Florio	Universit y of Avigon, France
Stéphane Gaubert	INRIA et Ecole polytechnique, France
Diogo Gomes	KAUST University, Saudi Arabia
Umberto Hryniewicz	UFRJ, Brasil and Ruhr-Universitt Bochum, Germany
Vincent Humilière	University Pierre et Marie Curie, France
Kei Irie	RIMS, Kyoto Unviersity, Japan
Hitoshi Ishii	Waseda University, Japon
Vadim Kaloshin	University of Maryland, USA and ITS-ETH, Switzerland
Konstantin Khanin	University of Toronto, Canada
Stanislas Kupin	University of Bordeaux, France
Renaud Leplaideur	University of Bretagne Occidentale, France
Olivier Ley	IRMAR - INSA de Rennes, France
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