Compact operators that commute with a contraction

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Abstract. Let T be a C_0–contraction on a separable Hilbert space. We assume that $I_H - T^*T$ is compact. For a function f holomorphic in the unit disk $\mathbb{D}$ and continuous on $\overline{\mathbb{D}}$, we show that $f(T)$ is compact if and only if f vanishes on $\sigma(T) \cap \mathbb{T}$, where $\sigma(T)$ is the spectrum of T and $\mathbb{T}$ the unit circle. If f is just a bounded holomorphic function on $\mathbb{D}$, we prove that $f(T)$ is compact if and only if $\lim_{n\to\infty} \|T^n f(T)\| = 0$.

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1. Introduction

Let H be a separable Hilbert space, and $\mathcal{L}(H)$ the space of all bounded operators on H. For $T \in \mathcal{L}(H)$, we denote by $\sigma(T)$ the spectrum of T. The Hardy space H^∞ is the set of all bounded and holomorphic functions on $\mathbb{D}$.

A contraction T on H is called a C_0–contraction (or in class C_0) if it is completely nonunitary and there exists a nonzero function $\theta \in \mathrm{H}^\infty$ such that $\theta(T) = 0$. A contraction T is said essentially unitary if $I_H - T^*T$ is compact, where I_H is the identity map on H.

Let T be a C_0–contraction on H, and let $\mathrm{H}^\infty(T) = \{f(T) : f \in \mathrm{H}^\infty\}$ be the subspace of the commutant $\{T\}' = \{A \in \mathcal{L}(H) : AT = TA\}$ obtained from the Nagy–Foias functional calculus. In this note we study the question of when $\mathrm{H}^\infty(T)$ contains a nonzero compact operator. B. Sz–Nagy [12] proved that $\{T\}'$ contains always a nonzero compact operator, but there exists a C_0–contraction T such that zero is the unique compact operator contained in $\mathrm{H}^\infty(T)$. Nordgreen [15] proved that if T is an essentially unitary C_0–contraction then $\mathrm{H}^\infty(T)$ contains a nonzero compact operator. There are also results about the existence of smooth operators (finite rank, Schatten–von Neuman operators) in $\mathrm{H}^\infty(T)$ (see [17]). It is also shown

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in Atzmon's paper [2], that if T is a cyclic completely nonunitary contraction such that $\sigma(T)=\{1\}$ and

$$\log\|T^{-n}\| = O(\sqrt{n}),\ n\to\infty, \tag{1}$$

then $T-I_H$ is compact.

Let $\mathcal{A}(\mathbb{D})$ be the usual disc algebra, i.e. the space of all functions which are holomorphic on $\mathbb{D}$ and continuous on $\overline{\mathbb{D}}$. In section 2 we study the compactness of $f(T)$ when f is in the disk algebra. We show (Corollary 2.3), that, if $f\in\mathcal{A}(\mathbb{D})$ and if T is a C_0–contraction which is essentially unitary, then $f(T)$ is compact if and only if f vanishes on $\sigma(T)\cap\mathbb{T}$. The main tool used in the proof of this result is the Beurling-Rudin theorem about the characterization of the closed ideals of $\mathcal{A}(\mathbb{D})$. We show also for a large class of C_0–contractions that the condition " T is essentially unitary" is necessary in the above result (Proposition 2.5) . As a consequence, we obtain that if T is a contraction that is annihilated by a nonzero function in $\mathcal{A}(\mathbb{D})$ and if T is cyclic (or, more generally, of finite multiplicity) then $f(T)$ is compact whenever $f\in\mathcal{A}(\mathbb{D})$ and f vanishes on $\sigma(T)\cap\mathbb{T}$. We notice that an invertible contraction with spectrum reduced to a single point and satisfying condition (1) is necessarily annihilated by a nonzero function in $\mathcal{A}(\mathbb{D})$ (see [1]).

In section 3, we are interested in the compactness of $f(T)$ when $f\in \mathrm{H}^\infty$. With the help of the corona theorem, we show (Theorem 3.4) that if T is an essentially unitary C_0-contraction, then $f(T)$ $(f\in\mathrm{H}^\infty)$ is compact if and only if $\lim\limits_{n\to\infty}\|T^nf(T)\|=0$. We obtain in particular that if $\lim\limits_{r\to1-} f(rz)=0$ for every $z\in\sigma(T)\cap\mathbb{T}$, then $f(T)$ is compact.

2. Compactness of $f(T)$ with f in the disk algebra

Let T be a contraction on H. We will introduce some definitions and results we will need later. We call $\lambda\in\sigma(T)$ a normal eigenvalue if it is an isolated point of $\sigma(T)$ and if the corresponding Riesz projection has finite rank. We denote by $\sigma_{np}(T)$ the set of all normal eigenvalues of T. The weakly continuous spectrum of T is defined by $\sigma_{wc}(T)=\sigma(T)\setminus\sigma_{np}(T)$ (see [14], p. 113). Let us suppose that T is essentially unitary and $\mathbb{D}\setminus\sigma(T)\neq\emptyset$. There exists a unitary operator U and a compact operator K such that $T=U+K$ and then we have $\sigma_{wc}(T)=\sigma_{wc}(U)\subset\mathbb{T}$ (see [5], [7] Theorem 5.3, p. 23 and [14] p. 115). It follows from the above observation that if $\mathbb{D}\setminus\sigma(T)\neq\emptyset$ then T is essentially unitary if and only if T^* is essentially unitary too.

Let $\mathcal{I}$ be a closed ideal of $\mathcal{A}(\mathbb{D})$. We denote by $S_{\mathcal{I}}$ the inner factor of $\mathcal{I}$, that is the greatest inner common divisor of all nonzero functions in $\mathcal{I}$ (see [8] p. 85). We set $Z(\mathcal{I})=\bigcap\limits_{f\in\mathcal{I}}\{\zeta\in\mathbb{T}: f(\zeta)=0\}$ and $\mathcal{J}(E)=\{f\in\mathcal{A}(\mathbb{D}): f_{|E}=0\}$, for $E\subset\mathbb{T}$. We shall need the Beurling-Rudin theorem [16] (see also [8] p. 85) about the structure of closed ideals of $\mathcal{A}(\mathbb{D})$, which states that every closed ideal $\mathcal{I}\subset\mathcal{A}(\mathbb{D})$ has the form

$$\mathcal{I}=S_{\mathcal{I}}\mathrm{H}^\infty\cap\mathcal{J}\big(Z(\mathcal{I})\big).$$

Theorem 2.1. *Let T be essentially unitary and $\mathbb{D} \setminus \sigma(T) \neq \emptyset$. If $f \in \mathcal{A}(\mathbb{D})$ and $f = 0$ on $\sigma(T) \cap \mathbb{T}$ then $f(T)$ is compact.*

For the proof of this theorem we need the following lemma.

Lemma 2.2. *Let T_1, T_2 be two contractions on H such that $T_1 - T_2$ is compact and $f \in \mathcal{A}(\mathbb{D})$. Then $f(T_1)$ is compact if and only if $f(T_2)$ is compact too.*

Proof. There exists a sequence $(P_n)_n$ of polynomials such that $\|f - P_n\|_\infty \to 0$, where $\|\cdot\|_\infty$ is the supremum norm on $\mathbb{T}$. For every n, $P_n(T_2) - P_n(T_1)$ is compact. By the von Neumann inequality, we have $\|(f - P_n)(T_i)\| \leq \|f - P_n\|_\infty$, $i = 1$ or 2. So $\|(f - P_n)(T_i)\| \to 0$ and

$$f(T_2) - f(T_1) = \lim_{n \to +\infty} \big(P_n(T_2) - P_n(T_1)\big).$$

Thus $f(T_2) - f(T_1)$ is compact. □

Proof of Theorem 2.1. Without loss of generality, we may assume that $\sigma(T) \cap \mathbb{T}$ is of Lebesgue measure zero. We set $\mathcal{I} = \{f \in \mathcal{A}(\mathbb{D})\colon f(T) \text{ compact}\}$; $\mathcal{I}$ is a closed ideal of $\mathcal{A}(\mathbb{D})$. We have to prove that $S_{\mathcal{I}} = 1$ and $Z(\mathcal{I}) \subset \sigma(T) \cap \mathbb{T}$. As observed above, we have $T = U + K$, where U is unitary and K is compact. Moreover, we have $\sigma_{wc}(U) = \sigma_{wc}(T) \subset \sigma(T) \cap \mathbb{T}$ ([14] p. 115), and since $\sigma_{np}(U)$ is countable, we see that $\sigma(U)$ is a subset of $\mathbb{T}$ of Lebesgue measure zero. By Fatou theorem ([8] p. 80), there exists a nonzero outer function $f \in \mathcal{A}(\mathbb{D})$ which vanishes exactly on $\sigma(U)$. Since U is unitary we have $f(U) = 0$. By Lemma 2.2, $f(T)$ is compact. This shows that $S_{\mathcal{I}} = 1$ and $Z(\mathcal{I}) \subset \sigma(U)$. We shall now show that $Z(\mathcal{I}) \subset \sigma_{wc}(U)$. Let $\lambda \in \sigma_{np}(U)$; λ is an isolated point in $\sigma(U)$ and $\mathrm{Ker}\,(U - \lambda I_H)$ is of finite dimension. There exists $f \in \mathcal{A}(\mathbb{D})$ with $f(\lambda) \neq 0$ and $f_{|\sigma(U) \setminus \{\lambda\}} = 0$. Since $(z - \lambda) f(z) = 0$ for every $z \in \sigma(U)$, and since U is unitary, $(U - \lambda I_H) f(U) = 0$ and $f(U)(H) \subset \mathrm{Ker}\,(U - \lambda I_H)$. So $f(U)$ is of finite rank, thus $f(U)$ is compact and by Lemma 2.2, $f(T)$ is compact. Hence $\lambda \notin Z(\mathcal{I})$. We deduce that $Z(\mathcal{I}) \subset \sigma_{wc}(U) \subset \sigma(T) \cap \mathbb{T}$, which finishes the proof.

Corollary 2.3. *Let T be an essentially unitary C_0–contraction and let $f \in \mathcal{A}(\mathbb{D})$. Then $f(T)$ is compact if and only if $f = 0$ on $\sigma(T) \cap \mathbb{T}$.*

Proof. It follows from Theorem 2.1 that if f vanishes on $\sigma(T) \cap \mathbb{T}$ then $f(T)$ is compact. Let now $f \in \mathcal{A}(\mathbb{D})$ such that $f(T)$ be compact. Let $\mathcal{B}_T$ denote a maximal commutative Banach algebra that contains I_H and T. We have $\sigma(T) = \sigma_{\mathcal{B}_T}(T)$, where $\sigma_{\mathcal{B}_T}(T)$ is the spectrum of T in $\mathcal{B}_T$. Let $\lambda \in \sigma(T) \cap \mathbb{T}$. There exists a character χ_λ on $\mathcal{B}_T$ such that $\chi_\lambda(T) = \lambda$ and have

$$|f(\lambda)| = |\lambda^n f(\lambda)| = |\chi_\lambda(T^n f(T))| \leq \|T^n f(T)\|. \tag{2}$$

Since T is in class C_0, $T^n x \to 0$ whenever $x \in H$, (see [11] Proposition III.4.1). Thus for every compact set $C \subset H$,

$$\lim_{n \to \infty} \sup_{x \in C} \|T^n x\| = 0.$$

For $C = \overline{f(T)(\mathbb{B})}$, where $\mathbb{B} = \{x \in H : \|x\| \leq 1\}$, we get $\lim_{n\to\infty} \|T^n f(T)\| = 0$. Then it follows from (2) that $f(\lambda) = 0$. □

Let $T \in \mathcal{L}(H)$. The spectral multiplicity of T is the cardinal number given by the formula

$$\mu_T = \inf \ \text{card}\ L,$$

where card L is the cardinal of L and where the infimum is taken over all nonempty sets $L \subset H$ such that $\text{span}\{T^n L;\ n \geq 0\}$ is dense in H. Notice that $\mu_T = 1$ means that T is cyclic.

Corollary 2.4. *Let T be a contraction on H with $\mu_T < +\infty$. Assume that there exists a nonzero function $\varphi \in \mathcal{A}(\mathbb{D})$ such that $\varphi(T) = 0$. Then $f(T)$ is compact for every function $f \in \mathcal{A}(\mathbb{D})$ that vanishes on $\sigma(T) \cap \mathbb{T}$.*

Proof. There exists two orthogonal Hilbert subspaces H_u and H_0 that are invariant by T, such that $H = H_u \oplus H_0$, $T_u = T_{|H_u}$ is unitary and $T_0 = T_{|H_0}$ is completely nonunitary (see [11], Theorem 3.2, p. 9 or [13], p. 7). Then T_0 is clearly in class C_0 and we have $\mu_{T_0} < +\infty$. By Proposition 4.3 of [4], $I_{H_0} - T_0^* T_0$ is compact. Let $f \in \mathcal{A}(\mathbb{D})$, with $f_{|\sigma(T)\cap\mathbb{T}} = 0$. Since $\sigma(T_0) \subset \sigma(T)$, it follows from Theorem 2.1 that $f(T_0)$ is compact. Now, since T_u is unitary and $\sigma(T_u) \subset \sigma(T) \cap \mathbb{T}$, we get $f(T_u) = 0$. Thus $f(T)$ is compact. □

Remark. Let T be a cyclic contraction satisfying condition (1) and with finite spectrum, $\sigma(T) = \{\lambda_1, \cdots, \lambda_n\} \subset \mathbb{T}$. By Theorem 2 of [1], there exists analytic function $f = \sum_{n\geq 0} a_n z^n$, $f \neq 0$, such that $\sum_n |a_n| < +\infty$ and $f(T) = 0$. Then, it follows from Corollary 2.4 that $(T - \lambda_1 I_H)\cdots(T - \lambda_n I_H)$ is compact. Thus we obtain a new proof Corollary 4.3 of [2], mentioned in the introduction.

Now we conclude this section by showing that the hypothesis "essentially unitary“ in Theorem 2.1 and Corollary 2.3 is necessary for a large class of contractions. Let us first make some observations. An operator $T \in \mathcal{L}(H)$ is called essentially normal if $TT^* - T^*T$ is compact, see [5]. Notice that if T is a C_0–contraction which is essentially unitary then T^* is essentially unitary too. Hence T is essentially normal since $I_H - T^*T$ and $I_H - TT^*$ are both compacts.

Proposition 2.5. *Let $T \in \mathcal{L}(H)$ be a C_0–contraction which is essentially normal and such that $\sigma(T) \cap \mathbb{T}$ is of Lebesgue measure zero. Assume that $f(T)$ is compact for every $f \in \mathcal{A}(\mathbb{D})$ vanishing on $\sigma(T) \cap \mathbb{T}$. Then T is essentially unitary.*

Proof. Let $\mathcal{K}(H)$ be the ideal of compact operators on H and $\pi : \mathcal{L}(H) \to \mathcal{L}(H)/\mathcal{K}(H)$ be the canonical surjection. The essential spectrum $\sigma_{ess}(T)$ of T is defined as the spectrum of $\pi(T)$ in the Banach algebra $\mathcal{L}(H)/\mathcal{K}(H)$. By Fatou theorem [8], there exists a non zero outer function $f \in \mathcal{A}(\mathbb{D})$ such that $f_{|\sigma(T)\cap\mathbb{T}} = 0$. By hypothesis $f(T)$ is compact. Let $\lambda \in \mathbb{D}$, the functions $z - \lambda$ and f have no common zero in $\overline{\mathbb{D}}$. So there exists two functions g_1 and g_2 in $\mathcal{A}(\mathbb{D})$ such that $(z - \lambda)g_1 + fg_2 = 1$. Thus $(T - \lambda I_H)g_1(T) + f(T)g_2(T) = I_H$, which shows that

$\pi(T) - \lambda\pi(I_H)$ is invertible in $\mathcal{L}(H)/\mathcal{K}(H)$. Hence $\sigma_{ess}(T) \subset \sigma(T) \cap \mathbb{T}$. By Rudin-Carleson-Bishop theorem (see [8] p. 81), there exists a function $h \in \mathcal{A}(\mathbb{D})$ such that $\overline{z} = h(z),\ z \in \sigma(T) \cap \mathbb{T}$. Since $\pi(T)$ is a normal element in the C^*–algebra $\mathcal{L}(H)/\mathcal{K}(H)$, we get $\pi(T)^* = h(\pi(T))$. On the other hand we have $1 - h(z)z = 0$ on $\sigma(T) \cap \mathbb{T}$, which implies that $\pi(I_H) - \pi(T)^*\pi(T) = \pi(I_H) - h(\pi(T))\pi(T) = 0$. Therefore $I_H - T^*T$ is compact. □

3. The case of $f(T)$ for $f \in \mathrm{H}^\infty$

In this section we are interested in the compactness of $f(T)$ when $f \in \mathrm{H}^\infty$. The spectrum of an inner function θ is defined by

$$\sigma(\theta) = \operatorname{clos} \theta^{-1}(0) \cup \operatorname{supp} \mu,$$

where μ is the singular measure associated to the singular part of θ and $\operatorname{supp} \mu$ is the closed support of μ (see [13], p. 63). Notice that for a C_0–contraction T on H, there exists a minimal inner function m_T that annihilates T, i.e $m_T(T) = 0$, and we have $\sigma(T) = \sigma(m_T)$, (see [11, 13]). As a consequence of Corollary 2.3 we prove the following result which was first established by Moore–Nordgren in [9], Theorem 1. The proof given in [9] uses a result of Muhly [10]. We give here a simple proof.

Lemma 3.1. *Let T be an essentially unitary C_0–contraction on H, and let θ be an inner function that divides m_T (i.e $m_T/\theta \in \mathrm{H}^\infty$) and such that $\sigma(\theta) \cap \mathbb{T}$ is of Lebesgue measure zero. Let $\psi \in \mathcal{A}(\mathbb{D})$ be such that $\psi_{|\sigma(\theta)\cap\mathbb{T}} = 0$. If $\phi = \psi m_T/\theta$, then $\phi(T)$ is compact.*

In particular the commutant $\{T\}'$ contains a nonzero compact operator.

Proof. Let $\Theta = m_T/\theta$ and $T_1 = T|_{H_1}$ be the restriction of T to $H_1 := \overline{\Theta(T)H}$; T_1 is a C_0–contraction with $m_{T_1} = \theta$. Moreover $I_{H_1} - T_1^*T_1 = P_{H_1}(I_H - T^*T)_{|H_1}$ is compact, where P_{H_1} is the orthogonal projection from H onto H_1. By Corollary 2.3, $\psi(T_1)$ is compact and thus $\phi(T) = \psi(T)\Theta(T) = \psi(T_1)\Theta(T)$ is also compact. □

Lemma 3.2. *Let T be an essentially unitary C_0–contraction on H, and let θ be an inner function that divides m_T and such that $\sigma(\theta) \cap \mathbb{T}$ is of Lebesgue measure zero. Let $f \in \mathrm{H}^\infty$ be such that $\lim\limits_{n\to+\infty} T^n f(T) = 0$. If $\phi = f m_T/\theta$, then $\phi(T)$ is compact.*

Proof. By the Rudin-Carleson-Bishop theorem, for every nonnegative integers n, there exists $h_n \in \mathcal{A}(\mathbb{D})$ such that $\overline{z}^n = h_n(z), z \in \sigma(\theta) \cap \mathbb{T}$ and $\|h_n\|_\infty = 1$, where $\|.\|_\infty$ is the supremum norm on $\mathbb{T}$ (see [8] p. 81). We have, for every n, $1 - z^n h_n(z) = 0,\ z \in \sigma(\theta) \cap \mathbb{T}$, then by Lemma 3.1, $(I_H - T^n h_n(T))\big(m_T/\theta\big)(T)$ is compact. So $\phi(T) - T^n f(T) h_n(T)\big(m_T/\theta\big)(T)$ is also compact. Since

$$\|T^n f(T) h_n(T)\big(m_T/\theta\big)(T)\| \le \|T^n f(T)\| \longrightarrow 0,$$

we deduce that $\phi(T)$ is compact. □

We need the following lemma about inner functions, which is actually contained in the proof of the main result of [15]. For the completeness we include here its proof.

Lemma 3.3. *Let Θ be an inner function. There exists a sequence $(\theta_n)_n$ of inner functions such that for each n, θ_n divides Θ, $\sigma(\theta_n)\cap\mathbb{T}$ is of Lebesgue measure zero and for every $z\in\mathbb{D}$, $\lim_{n\to+\infty}\theta_n(z)=\Theta(z)$.*

Proof. Let B_n be the Blaschke product constructed with the zeros of Θ contained in the disk $\{|z|\le 1-1/n\}$, each zero of Θ repeated according to its multiplicity. Let ν be the singular measure defining the singular part of Θ. There exists $F\subset\mathbb{T}$ of Lebesgue measure zero such that $\nu(F)=\nu(\mathbb{T})$. There exists a sequence $(K_n)_n$ of compact subsets of F such that $\lim_{n\to\infty}\nu(K_n)=\nu(F)$. For every n, let ν_n be the measure on $\mathbb{T}$ defined by $\nu_n(E)=\nu(E\cap K_n)$. Denote by S_n the singular inner function associated to the measure ν_n. We only need now to take $\theta_n=B_nS_n$. □

We are now able to prove the main result of this section.

Theorem 3.4. *Let T be an essentially unitary C_0–contraction on H. Let $f\in\mathrm{H}^\infty$. Then the following assertions are equivalent.*

(1) $\lim_{n\to\infty}\|T^nf(T)\|=0$,
(2) $f(T)$ is compact.

Proof. (1) $\Rightarrow$ (2) : Let $\Theta=m_T$ and let $(\theta_n)_n$ be the sequence of inner functions given by Lemma 3.3. For every n, we set $\varphi_n=m_T/\theta_n$. Since $(\varphi_n)_n$ is a bounded sequence in H^∞ and $\varphi_n(z)\longrightarrow 1$ $(z\in\mathbb{D})$, $(\varphi_n)_n$ converges to 1 uniformly on compact subsets of $\mathbb{D}$. Then, for every k, there exists a nonnegative integer n_k such that $|\varphi_{n_k}(z)|\ge e^{-1}$ for $|z|\le k/(k+1)$. Clearly the sequence $(n_k)_k$ may be chosen to be strictly increasing. Moreover for $|z|\ge k/(k+1)$, we have $|z^k|\ge e^{-1}$. So

$$e^{-1}\le|z^k|+|\varphi_{n_k}(z)|\le 2,\ z\in\mathbb{D}.$$

By he corona theorem ([13], p. 66), there exists two functions $h_{1,k}$ and $h_{2,k}$ in H^∞ such that

$$z^kh_{1,k}+\varphi_{n_k}h_{2,k}=1\ \text{ and }\ |h_{1,k}|,|h_{2,k}|\le C,$$

where C is an absolute constant. Thus we get

$$T^kf(T)h_{1,k}(T)+f(T)\varphi_{n_k}(T)h_{2,k}(T)=f(T),$$

and

$$\|T^kf(T)h_{1,k}(T)\|\le C\|T^kf(T)\|\longrightarrow 0.$$

Consequently, $f(T)=\lim_{k\to\infty}f(T)\varphi_{n_k}(T)h_{2,k}(T)$ in the $\mathcal{L}(H)$ norm. Finally $f(T)$ is compact since by Lemma 3.2, for every k, $f(T)\varphi_{n_k}(T)h_{2,k}(T)$ is compact.

(2) $\Rightarrow$ (1) : see the proof of Corollary 2.3. □

As in Corollary 2.4, Theorem 3.4 holds for a C_0-contraction with $\mu_T < +\infty$.

Let T be a contraction on H. It is shown by Esterle, Strouse and Zouakia in [6], that if $f \in \mathcal{A}(\mathbb{D})$, then $\lim_{n\to\infty} \|T^n f(T)\| = 0$ if and only if f vanishes on $\sigma(T) \cap \mathbb{T}$. So Theorem 3.4 implies Corollary 2.3. Now, if T is completely non unitary, Bercovici showed in [3] that if $f \in \mathrm{H}^\infty$ and $\lim_{r\to 1-} f(rz) = 0$, for every $z \in \sigma(T) \cap \mathbb{T}$, then $\lim_{n\to\infty} \|T^n f(T)\| = 0$. So it follows immediately from this fact and Theorem 3.4 the following result.

Corollary 3.5. *Let T be an essentially unitary C_0–contraction on H. Let $f \in \mathrm{H}^\infty$. If for every $z \in \sigma(T) \cap \mathbb{T}$, $\lim_{r\to 1-} f(rz) = 0$, then $f(T)$ is compact.*

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